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Analyzing Bangladeshi Participation Rates by Age Group using Mathematical Modeling

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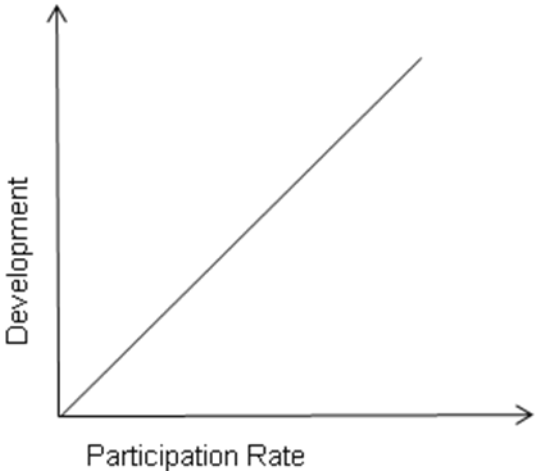
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Abstract: *The primary objective of this research is to fit mathematical models to the forward cumulative distribution of age-specific participation rates (ASPRs) for the male, female, and mixed-sex populations of Bangladesh. The 2011 Statistical Year Book of Bangladesh is the source for this data. In order to fit these mathematical models, the STATISTICA program makes use of the Quasi Newton Method. In addition, the model's accuracy and validity are checked using t-tests, F-tests, and cross validity prediction power (CVPP). The results show that ASPRs use a cubic polynomial model for their forward cumulative distribution and a third-degree polynomial model for the population as a whole, regardless of gender. The t-test, the F-test, and the cross validity prediction power all show that these models are well-fit. More than 95%, 84%, 96%, 99%, and 99% of these models are stable, respectively.*

Keywords: *Age specific participation rates (ASPRs), Polynomial model, t-test, F-test, Cross validity prediction power (CVPP).*

1. INTRODUCTION

The ASPR is calculated by dividing the number of people in the labor force by the population of a certain age. The rate of involvement in the labor force is being referred to here. One way to measure the size of Bangladesh's labor force is to look at the percentage of the population that is working-age. The percentage of the population actively seeking gainful employment determines the wealth distribution in any nation. However, ASPR by sex is a great tool for predicting how a country's workforce will evolve in the future. From 1961 to 2001, there was an upward trend in the participation rate in Bangladesh [1]. While men's involvement rates tend to be higher across all age groups, women's rates tend to be lower for a variety of reasons, such as lower levels of education, less safe housing options, a higher incidence of female labor force separation, and so on. For a country's progress as a whole, it is possible to raise the participation rate of women by taking the right measures. The correlation between a country's participation rate and its level of development is inevitable. Here, progress in any nation's population is progress in all spheres, including the social, economic, environmental, educational, and, most importantly, health spheres. However, a nation's economic solvency is a direct or indirect determinant of any kind of progress. It has been noted that a high participation rate leads to high development, which in turn leads to an upward straight line.



One way to measure the financial stability of a society is to look at the female participation rate by age group. So, following, we address the study's objectives: i) analyze the distribution and trend of age-specific participation rates in Bangladeshi men, women, and transgender populations; ii) apply mathematical models to the data of ASPRs and forward cumulative ASPRs in 2010 for all three genders in Bangladeshi populations; and iii) validate these models by test-taking with CVPP.

2. Data and Data Source

A secondary data on ASPRs for male, female and both sexes population in Bangladesh have been taken from Statistical Year Book of Bangladesh of 2011 [2] which is shown in Table 1 and their forward cumulative distribution are demonstrated in Table 2.

Table 1. Age Specific Participation Rate by Sex, 2010

Age Group	Central Age	Male	Female	Both Sex
15-19	17.5	48.44	29.40	39.37
20-24	22.5	75.93	40.98	56.70
25-29	27.5	92.19	44.71	66.61
30-34	32.5	97.29	46.62	70.80
35-39	37.5	98.34	47.67	72.81
40-44	42.5	98.05	46.25	72.82
45-49	47.5	97.37	47.58	74.28
50-54	52.5	94.11	10.25	56.07
55-59	57.5	88.52	11.18	51.91
60-64	62.5	77.20	6.63	45.03
65+	67.5	57.93	8.32	34.77

Table 2. Forward Cumulative Distribution of Age Specific Participation Rates by Sex, 2010

Age Group	Central Age	FCD of Male	FCD of Female	FCD of Both Sex
15-19	17.5	48.44	29.4	39.37
20-24	22.5	124.37	70.38	96.07
25-29	27.5	216.56	115.09	162.68
30-34	32.5	313.85	161.71	233.48
35-39	37.5	412.19	209.38	306.29
40-44	42.5	510.24	255.63	379.11
45-49	47.5	607.61	303.21	453.39
50-54	52.5	701.72	313.46	509.46
55-59	57.5	790.24	324.64	561.37
60-64	62.5	867.44	331.27	606.4
65+	67.5	925.37	339.59	641.17

Note: FCD: Forward Cumulative Distribution

3. METHODS AND METHODOLOGICAL ISSUES

3.1 Model Fitting

A) Using the scattered plot (Fig.1-Fig.3) of ASPRs by age group in years for male, female and both sexes population in Bangladesh, it seems that ASPRs for male, female and both sexes population in Bangladesh can be fitted by polynomial model with respect to different ages in year. Therefore, an nth degree polynomial model is considered and the form of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i \; x^i + u;$$

where, x is the mean value of the age group; y is ASPRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the disturbance term of the model. Here, a suitable n is found for which the error sum of square is minimum.

B) Using the dotted plot (Fig.4-Fig.6) of forward cumulative ASPRs for male, female and both sexes population in Bangladesh by age groups, it is observed that these follow nth degree polynomial model with respect to ages. Therefore, the structure of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i \; x^i + u;$$

where, x is the average value of the age group; y is forward cumulative ASPRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the disturbance term of the model. A suitable n is selected such that the error sum of square is lowest.

The software STATISTICA was used to fit these mathematical models to ASPRs.

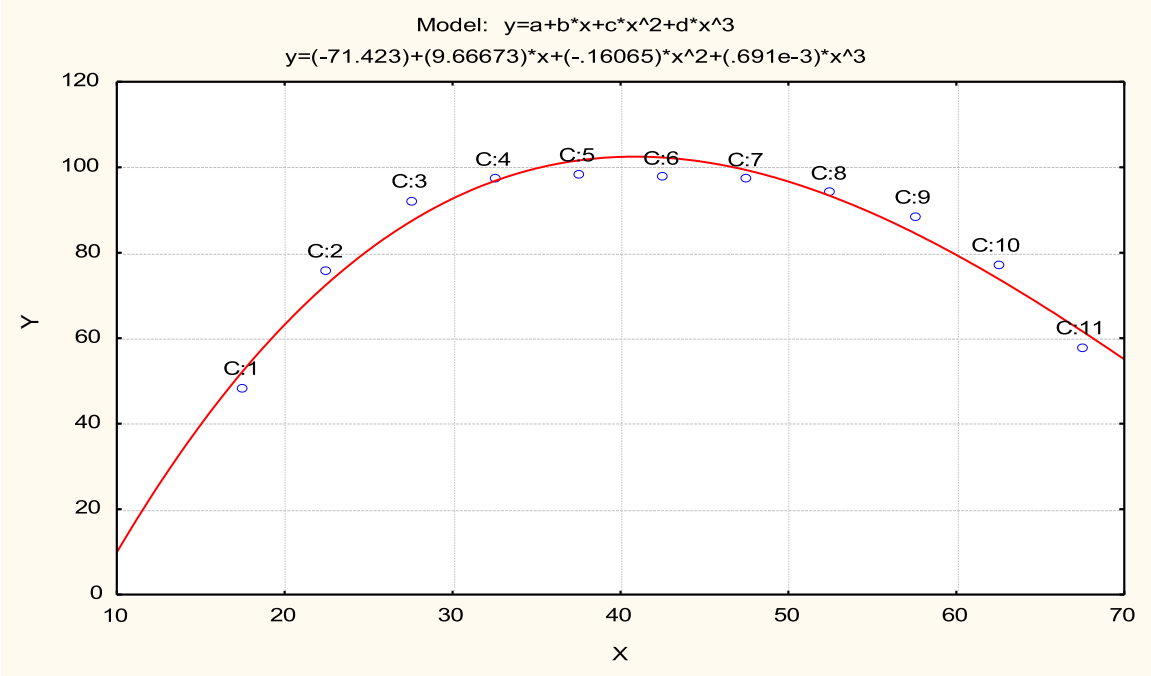


Fig.1. Observed and Fitted ASPRs for Male in 2010. X: Age in Years and Y: ASPRs for Male.

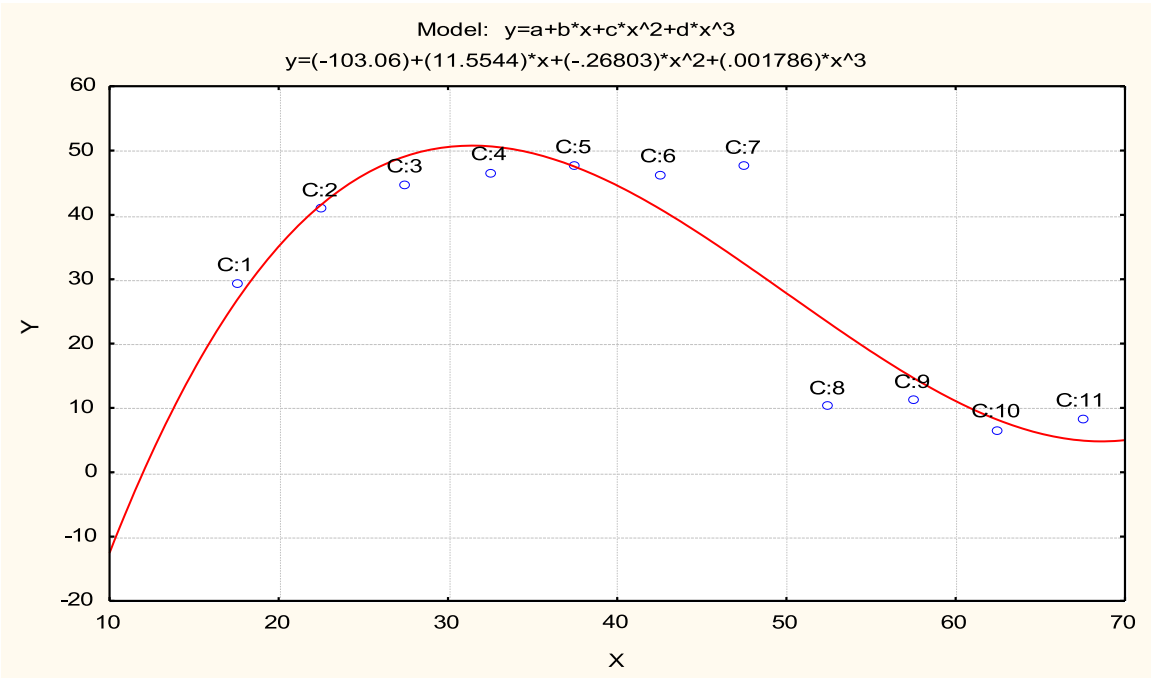


Fig.2. Observed and Fitted ASPRs for Female in 2010. X: Age in Years and Y: ASPRs for Female.

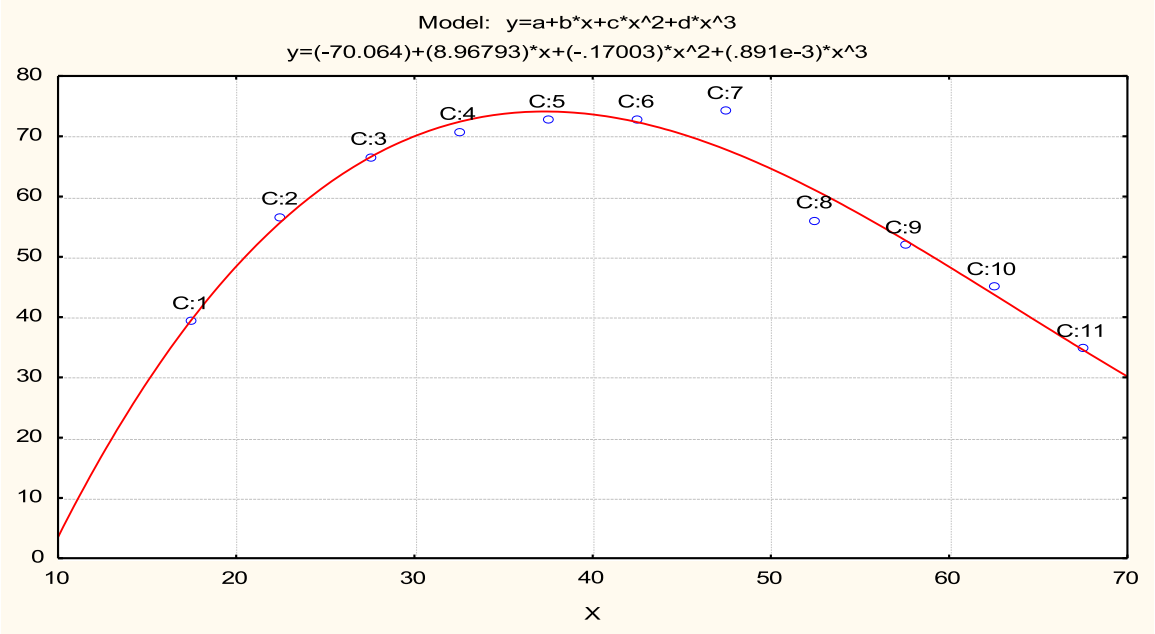


Fig.3. Observed and Fitted ASPRs for Both Sexes in 2010. X: Age in Years and Y: ASPRs for Both Sexes.

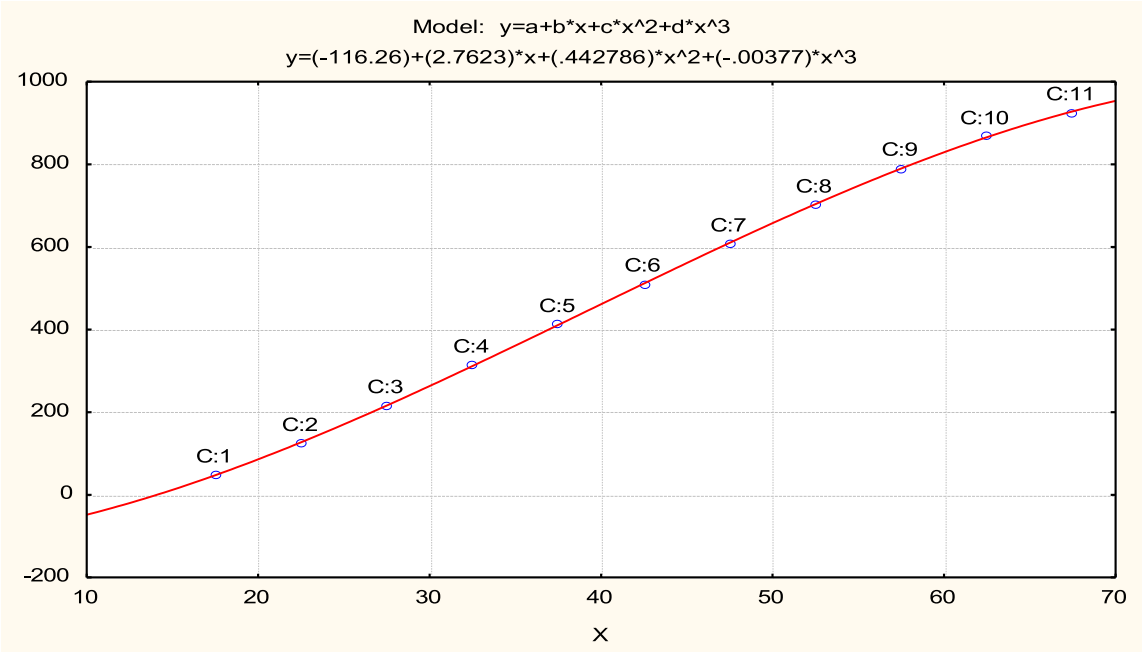


Fig.4. Observed and Fitted FCD of ASPRs for Male in 2010. X: Age in Years and Y: FCD of ASPRs for Male.

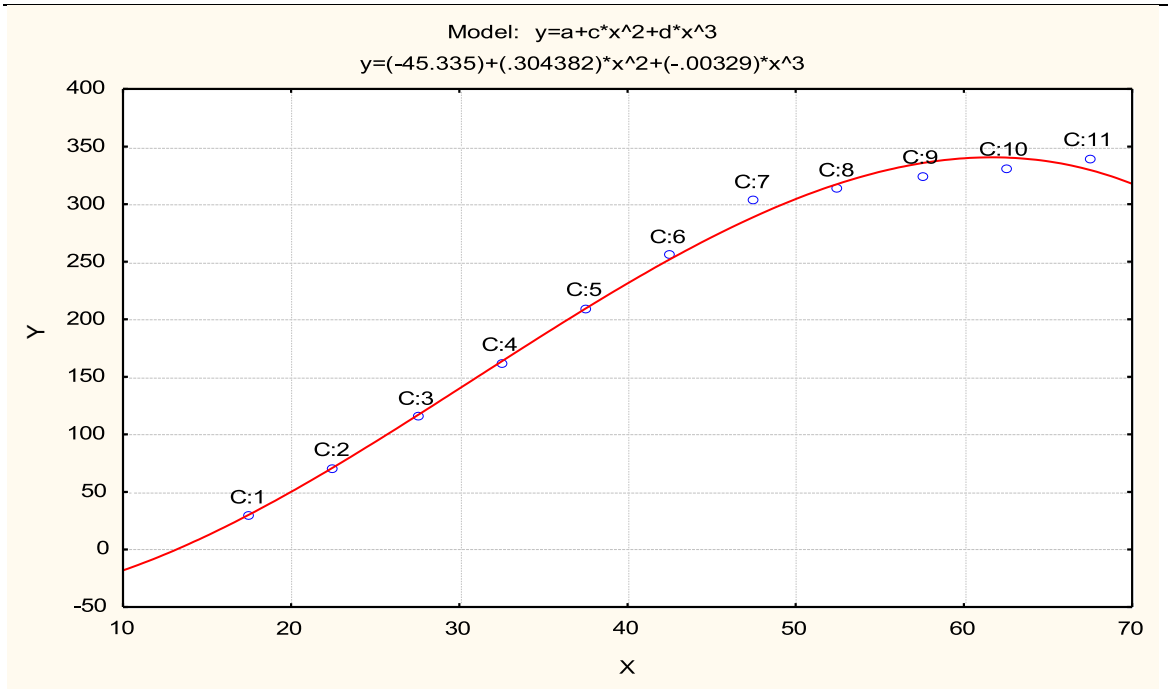


Fig.5. Observed and Fitted FCD of ASPRs for Female in 2010. X: Age in Years and Y: FCD of ASPRs for Female.

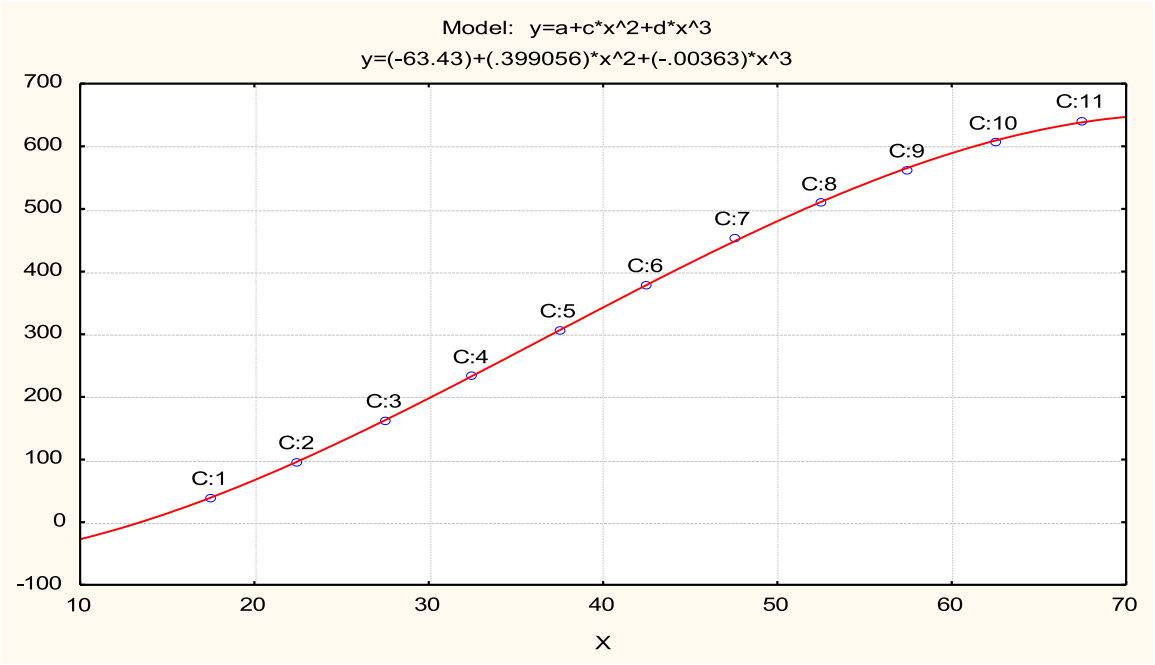


Fig.6. Observed and Fitted FCD of ASPRs for Both Sexes in 2010. X: Age in Years and Y: FCD of ASPRs for Both Sexes.

3.2 Model Validation Technique

To check how much these models are stable over the population, the cross validity prediction power (CVPP), ρ^2_{cv}*, is applied. Here*

$\rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$; where, n is the number of cases, k is the number of predictors in the model and the cross validated R is the correlation between

observed and predicted values of the dependent variable [3] (Stevens, 1996). The shrinkage

coefficient of the model is the positive value of (ρ^2 - R^2); whereρ^2 is CVPP and R^2 is the coefficient of determination of the model. 1-shrinkage is the stability of R^2 of the model. The information on model fitting and estimated CVPP has been demonstrated in Table 3. Note that CVPP was also employed as model validation or accuracy test [4, 5, 6, 7, 8].

3.3 F-test

To verify the measure of the overall significance of the model as well as the significance of R^2 , the F-test is employed here. The formula for F-test is given below:

$$F = \frac{R^2 / (k-1)}{(1-R^2) / (n-k)}$$

where k = the number of parameters is to be estimated, n = the number of cases and $\mathbf{R^2}$ is the coefficient of determination in the model [9]. These estimates are shown in Table 4.

4. Results and Discussion

The ASPRs for the male, female, and mixed-sex populations of Bangladesh in 2010 are shown in Table 1. Figure 7 displays the data displayed on graph paper, allowing one to see the amount and pattern of ASPRs for both the male and female population of Bangladesh. A male ASPR pattern that is the inverse of a female U-shape pattern has been discovered. The male ASPR ranges from 98.34 in the 35–39 age group to 57.93 in the 65+ age group. Additionally, the age groups of 35–39 and 60–64 years old had the lowest ASPRs at 6.63 and 46.67, respectively. In contrast to males, females in Bangladesh have a much lower involvement rate across all age groups (Fig. 7). Table 2 displays the total ASPRs for each age group and is calculated for the forward cumulative distribution of the male, female, and both sex population of Bangladesh.

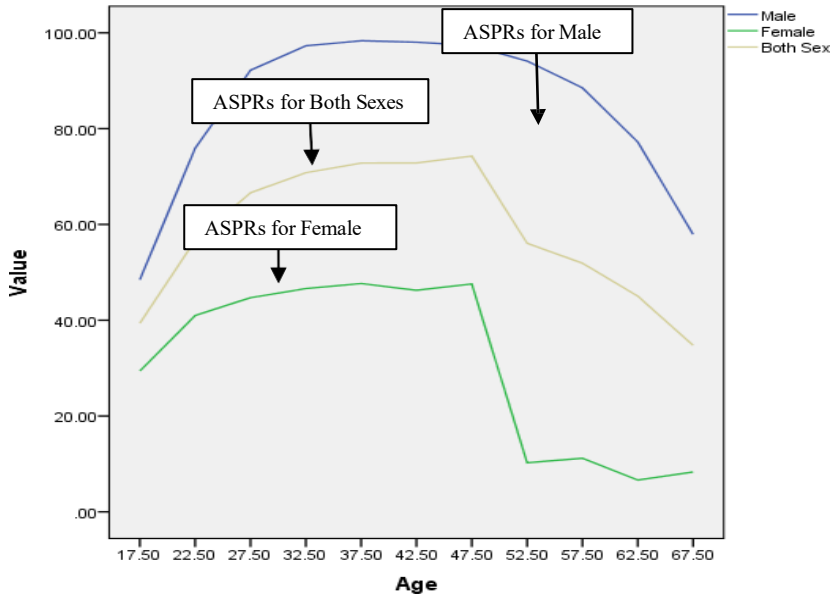


Fig.7. ASPRs for male, female and both sexes population of Bangladesh in 2010.

The polynomial model is constructed for ASPRs for male population of Bangladesh in 2010 and the fitted equation is as follows:

$$y = -71.423 + 9.66673x - 0.16065x^2 + 0.000691x^3 \quad (1)$$

t-stat	(-2.7012)	(4.4455)	(-2.9425)	(1.6222)
P-value	(0.0305)	(0.0029)	(0.0216)	(0.1487)

Again, another polynomial model is constructed for ASPRs for female population of Bangladesh in 2010 and the fitted equation is

$$y = -103.06 + 11.5544x - 0.26803x^2 + 0.001786x^3 \quad (2)$$

t-stat	(-1.9386)	(2.6427)	(-2.4416)	(2.0855)
P-value	(0.0937)	(0.0332)	(0.0446)	(0.0754)

Another polynomial model is fitted for ASPRs for both sexes population of Bangladesh in 2010 and the fitted equation is given below:

$$y = -70.0639 + 8.967925x - 0.170033x^2 + 0.000891x^3 \quad (3)$$

t-stat	(-3.3931)	(5.2809)	(-3.9879)	(2.6804)
P-value	(0.0115)	(0.0011)	(0.0052)	(0.0315)

Moreover, another polynomial model is constructed for forward cumulative distribution of ASPRs for male population of Bangladesh in 2010 and the fitted equation is as follows:

$$y = -116.262 + 2.7623x + 0.442786x^2 - 0.003774x^3 \quad (4)$$

t-stat	(-8.4760)	(2.4487)	(15.6338)	(-17.0820)
P-value	(0.0000)	(0.0441)	(0.0000)	(0.0000)

Again, another polynomial model is fitted for forward cumulative distribution of ASPRs for female population of Bangladesh in 2010 and the fitted equation is

$$y = -45.3354 + 0.304382x^2 - 0.003290x^3 \quad (5)$$

t-stat	(-5.4938)	(22.4983)	(-17.3108)
P-value	(0.0005)	(0.0000)	(0.0000)

In which the term containing x is excluded from the model because of the insignificant of the parameter.

Furthermore, another polynomial model is constructed for forward cumulative distribution of ASPRs for both sexes population of Bangladesh in 2010 and the fitted equation is given by

$$y = -63.4298 + 0.399056x^2 - 0.003631x^3 \quad (6)$$

t-stat	(-21.4841)	(82.4435)	(-53.3971)
P-value	(0.0000)	(0.0000)	(0.0000)

In (6), the term containing x is kept out from the model because of the insignificant of the coefficient of x.

The estimated CVPP, ρ_{cv}^2 corresponding to their R^2 are shown in Table 3. The observed and fitted values are depicted in Figure 1 to Figure 6. In this table, all fitted models from equation (1) to equation (6) are highly cross validated and their shrinkage's are small. Moreover, it is observed that all the parameters of the fitted models are statistically significant with large proportion of variation explained excepting only one model and whose proportion of variation is explained more than 84%. The stability of R^2 of these models are more than 94%, 79%, 95%, 99%, 99%, and 99% respectively.

Table 3. Information on Model Fittings and Estimated CVPP of the Predicted Equations of ASPRs and its Forward Cumulative Distribution of Bangladesh in 2010.

Models	n	k	R^2	ρ_{cv}^2	Shrinkage	Variance explained (%)
Equation 1	11	3	0.95920	0.904623377	0.054576623	95.920
Equation 2	11	3	0.84762	0.643787013	0.203832987	84.762
Equation 3	11	3	0.96334	0.914301299	0.049038701	96.334
Equation 4	11	3	0.99996	0.999906494	0.000053506	99.996
Equation 5	11	2	0.99573	0.992513636	0.003216364	99.573
Equation 6	11	2	0.99984	0.999719481	0.000120519	99.984

The calculated values of F-statistic of the models (1) to (6) are 1152.947 with (3, 7) degrees of freedom (d.f.), 46.77013 with (3, 7) degrees of freedom (d.f.), 921.1711 with (3, 7) degrees of freedom (d.f.), 195734.8 with (3, 7) degrees of freedom (d.f.), 3302.224 with (2, 8) degrees of freedom (d.f.), 72223.45 with (2, 8) degrees of freedom (d.f.) respectively where as the corresponding tabulated values are only 8.45, 8.45, 8.45, 8.45, 8.65 and 8.65 at 1% level of significance, respectively. Therefore, from these statistics it is also concluded that all these constructed models are highly statistically significant. Hence, the fits of all these models are well.

Table 4. Information on Model Fittings and Calculated Tab. F of the Predicted Equations of ASPRs and its Forward Cumulative Distribution of Bangladesh in 2010.

Models	n	k	Cal. F	Tab.F (at 1% level)
Equation 1	11	4	1152.947	8.45 with (3,7) d.f.
Equation 2	11	4	46.77013	8.45 with (3,7) d.f.
Equation 3	11	4	921.1711	8.45 with (3,7) d.f.
Equation 4	11	4	195734.8	8.45 with (3,7) d.f.
Equation 5	11	3	3302.224	8.65 with (2,8) d.f.
Equation 6	11	3	72223.45	8.65 with (2,8) d.f.

5. CONCLUSIONS

This research found that compared to the ASPRs for males in Bangladesh, which had a U-shaped pattern, the female population's ASPRs had a very poor form. With a major percentage of the variation explained by cubic polynomial models, the ASPRs and their forward cumulative distribution follow suit. Over 95%, 84%, 96%, 99%, and 99% of these models are stable, respectively. In addition, R 2 values greater than 94%, 79%, 95%, 99%, and 99% for these models, respectively, indicate stability. According to the F-statistics, these models have a very high level of overall significance.

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