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Developing a novel mathematical framework for a cell-based manufacturing system

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Abstract

We provide a novel mathematical model for cell creation in cellular manufacturing systems (CMS) that is based on the idea of cell use in this research. The model's goal is to maximize cell utilization performance while minimizing exceptional elements (EE) and the amount of cell voids. In order to ensure that the suggested model can successfully group parts and machines, it is compared to other models in the literature and a number of test problems are carried out. These models include [S.J. Chen, C.S. Cheng, A neural network-based cell formation algorithm in cellular manufacturing, International Journal of Production Research 33 (2) (1995) 293-318], [I. Mahdavi, O.P. Kaushal, M. Chandra, Graph-neural network approach in cellular manufacturing based on a binary system, International Journal of Production Research 39 (13) 2913-2922].

Keywords: Cell formation; CMS; Mathematical model; Cell utilization

1. Introduction

Group technology (GT) was developed in the Soviet Union in the 1940s as a way to increase output from batch production systems ([3]). The majority of products are probably made via batch manufacturing. Improving the efficiency and output of batch production is becoming increasingly important. When numerous distinct components with low yearly quantities are manufactured in tiny lot sizes using a batch-flow manufacturing system, GT performs at its best. In order to construct a family of parts from a set of machines, it is necessary to arrange parts into part-families and machines into machine-cells. Both the machine-cell formation (MCF) and the machine-component grouping (MCG) problems involve the determination of machine-groups and part-families.

One way that the idea of group technology may be applied to the design of industrial systems is via cellular manufacturing. Machines are categorized into machine cells according to the value they provide to the production process; the fundamental premise of group technology is to find shared characteristics and processes in manufacturing.

Based on the tooling needs of the parts and the tooling that is available on the machines, Defersha and Chen [5] presented a thorough mathematical model for CMS design. The model takes into account a number of factors, such as dynamic cell configuration, alternative routings, lot splitting, sequence of operations, machine capacity, workload balancing among cells, operation cost, cost of subcontracting part processing, setup cost, tool consumption cost, cell size limits, and machine adjacency constraints. It also includes multiple units of identical machines.

For cellular manufacturing systems (CMSs) dealing with unpredictable demand, Tavakkoli-Moghaddam et al. [11] introduced a novel mathematical model for facility layout optimization. In machine and cell layout challenges, the goal is to reduce the overall costs of inter- and intra-cell motions at the same time.

To build these cells in real time, Slomp et al. [9] thought up a novel kind of virtual cellular manufacturing (CM) system and a multi-objective design approach. In order to reap the advantages of CM while keeping the functional architecture, virtual cells are used to temporarily group machines,

tasks, and workers. Depending on changes in demand quantities and mix, virtual cells are produced frequently, such as every week or every month, when new tasks accrue over a planning period. In addition to considering part-machine grouping, the technique also considered labor grouping. The process relies on goal programming methods that emphasize interaction. Considerations include, but are not limited to, limitations on capacity and cell size, reduction of load imbalances, minimizing of part movements between cells, supply of flexibility, and so on. Labor pooling involves dividing and reorganizing functionally specialized labor pools into virtual cells. Considerations such as minimizing of worker moves between cells, giving sufficient labor flexibility, guaranteeing balanced loads for workers, etc. are approached pragmatically.

A novel mathematical method for the formation of manufacturing cells was suggested by Albadawi et al. [2]. There are two parts to the suggested strategy. The first step is to use factor analysis on the similarity coefficient matrix to find the machine cells. The next step is to allocate components to the specified machine cells using an integer-programming approach.

To assess the compromise between competing goals in process plan selection and cell creation, Kizil and Ozbayrak [6] created an algorithm. Although it is essential, cellular manufacturing must also take into account the reduction of material transportation between cells if it is to create a system with a minimum overall labor content. It would be very laborious and expensive to solve the cell formation and process plan selection problems for every feasible alternative process plan.

For the purpose of designing cellular manufacturing systems, Wang [12] introduced a linear assignment method for machine-cell and part-family construction. Finding a collection of the least similar parts or machines and comparing their similarity coefficients is the first step in the current strategy to determining part-family or machine-cell representatives. A linear assignment model is developed to solve the formation issue by maximizing a similarity index and using the group representatives and their associated similarity coefficients to distribute the remaining parts or machines. A group creation method is created using the linear assignment model that has been described. The offered method is very efficient and successful, particularly when dealing with large-sized issues, according to a comparison research that used several performance metrics and numerous existing data sets.

A graph-neural network manufacturing strategy was suggested by Mahdavi et al. [7] to address cell creation challenges in group technology. An algorithm that outperforms more traditional approaches has been developed. In the presence of bottleneck machines and/or bottleneck parts, this study can address large-scale industrial challenges without assuming any parameters or unusual factors. For efficient and economical part family and machine cell generation, Taboun et al. [10] laid up a two-step process.

A mixed-integer mathematical model for machine grouping and part family assignment is first used to outline the issue. They call it the single-stage model, and it takes into account the importance of real-life planning factors including cell layout, machine procurement and salvage, subcontracting, inter-cell transportation, and capital expenditure in terms of cost trade-offs. They split this one-stage model into two parts to make it easier to compute: first, a heuristic for forming machine cells and part families; second, combining the heuristic with a mathematical program to optimize different cost characteristics.

In the context of cellular manufacturing, Chen and Cheng [4] investigated a cell generation method that relies on neural networks. As a new approach to the cell creation issue in GT, the adaptive resonance theory (ART) neural network has recently emerged. An ART network has several benefits over more traditional approaches, including its rapid processing and an exceptional aptitude for dealing with industrial challenges on a huge scale. One drawback of this method is that, particularly when dealing with bottleneck machines and/or parts, the quality of the grouping solution is quite sensitive to the initial arrangement of the machine-part incidence matrix. The primary goal of this study was to identify a set of supplemental methods that may help alleviate the difficulty highlighted before. Forty examples show how the supplemental processes are beneficial. The results show that their method outperforms the competition when faced with data that is not well-structured.

While designing a cellular manufacturing system, Adil et al. [1] took both the investment and operational expenses into account at the same time. In order to do this, they create a mixed-integer programming model and show how investment and operational variables like sequence dependent setup, machine idle time, part inventory, and part early and late completion (compared with due date) are related in a trade-off way.
When thinking about problems with cellularization portion and machine selection, a mathematical framework was given by Rajamani et al. [8]. The ideal number of parts, percentage of demand to create in cells, machinery to use, and production schedules may all be better determined with the help of the built models. For both relaxed and linear mixed integer algorithms, they provide efficient column generation strategies.
In Section 2, the problem formulation of the suggested model is described. The remainder of this study is arranged in the following way. Section 4 offers a discussion on comparing the acquired findings, whereas Section 3 includes numerical examples. Part 5 concludes with some last thoughts.

2. Problem formulation

Here, we use CMS machine-parts to develop a novel nonlinear mathematical model. In order to obtain better cell utilization efficiency, this model focuses on reducing the amount of exceptional elements (EE) and cell voids.
Multiplying the number of non-zero elements of the block-diagonal matrix by the cell's block-diagonal matrix size yields the cell utilization notion.

2.1. Indexing sets

i index for parts ($i = 1, 2, \ldots, P$),
j index for machines ($j = 1, 2, \ldots, M$),
k index for cells ($k = 1, 2, \ldots, C$).

2.2. Parameters

L_k: lower bound of the number of machines in cell k.
U_k: upper bound of the number of machines in cell k.
LP: lower bound of the number of parts in cells.

$$r_{ij} = \begin{cases} 1 & \text{if part } i \text{ needs to machine } j, \\ 0 & \text{otherwise.} \end{cases}$$

2.3. Decision variable

Y_{jk} Z_{ik}

$$= \begin{cases} 1 & \text{if machine } j \text{ is assigned to cell } k, \\ 0 & \text{otherwise,} \end{cases}$$

$$= \begin{cases} 1 & \text{if part } i \text{ is assigned to cell } k, \\ 0 & \text{otherwise,} \end{cases}$$

2.4. Objective function

We propose the objective function as:

$$\text{Min } Z = \sum_{k=1}^{C} \sum_{i=1}^{P} \sum_{j=1}^{M} Z_{ik} Y_{jk} - \sum_{i=1}^{P} \sum_{j=1}^{M} Z_{ik} Y_{jk} r_{ij} \quad (1)$$

The objective function accounts for calculation of total number of voids in all cells.

2.5. Constraints

Lower and upper bound of number of machines to be allocated to cells:

$$\sum_{j=1}^{M} Y_{jk} \geq L_k \quad \forall k, \quad (2)$$

$$\sum_{j=1}^{M} Y_{jk} \leq U_k \quad \forall k. \quad (3)$$

Each machine must be allocated to one cell:

$$\sum_{k=1}^{K} Y_{jk} = 1 \quad \forall j. \quad (4)$$

Each part must be assigned to one cell:

$$\sum_{k=1}^{K} Z_{ik} = 1 \quad \forall i. \quad (5)$$

Lower limit of number of parts in each cell:

$$\sum_{i=1}^{K} Z_{ik} \geq Lp \quad \forall k. \quad (6)$$

We could specify minimum utilization of cells to be achieved as follows:

$$\sum_{i=1}^{P} \sum_{j=1}^{M} Z_{ik} Y_{jk} r_{ij} \geq \min \text{ ut}_k \times \sum_{i=1}^{P} \sum_{j=1}^{M} Z_{ik} Y_{jk} \quad \forall k. \quad (7)$$

$$Z_{ik}, Y_{jk} \in \{0, 1\} \quad 6i, j, k. \tag{11}$$

2.6. Linearization of the mathematical model

It is obvious that Eqs. (1) and (7) are nonlinear functions. In fact, these functions can be linearized by defining new binary integer variable W_{ijk}, which is computed by the following equation:

$$W_{ijk} = Z_{ik}.Y_{jk} \quad 6i, j, k.$$

By considering the above equation, following equations should be added to the proposed model by enforcing these two linear inequalities simultaneously:

$$W_{ijk} — Z_{ik} — Y_{jk} + 1.5 \mathbf{P} 0 \quad 6i, j, k, \tag{12}$$

$$1.5W_{ijk} — Z_{ik} — Y_{jk} 6 0 \quad 6i, j, k. \tag{13}$$

3. Computational results

Tables 1 and 2 show the machine–part matrix to the examples consisting of 6 · 6 machine–part (Example 1), and 10 · 10 machine–part (Example 2) respectively.

To validate and verify the proposed model, two illustrative examples as mentioned from the literature are optimally solved by the Lingo 8 software using branch-and-bound (B&B) with respect to the presented parameter setting in Table 3.

Table 1
Chen and Cheng [4] – 6 · 6 machine–part matrix

	Parts					
	P1	P2	P3	P4	P5	P6
*M*1	0	1	0	1	0	1
*M*2	0	0	0	0	1	1
*M*3	0	1	0	0	1	1
*M*4	1	0	1	0	0	0
*M*5	1	0	1	0	0	0
*M*6	0	1	0	1	1	0

Table 2
Chen and Cheng [4] – 10 · 10 machine–part matrix

	Parts									
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
*M*1	1	1	0	0	0	1	0	0	0	0
*M*2	1	0	0	0	0	0	0	0	0	0
*M*3	1	0	0	0	0	1	0	0	0	0
*M*4	0	0	1	0	0	0	1	1	0	0
*M*5	0	0	1	0	0	0	0	1	0	0
*M*6	0	0	0	1	0	0	0	0	1	0
*M*7	0	0	0	1	1	0	0	0	1	1
*M*8	0	0	0	1	1	0	0	0	1	1
*M*9	1	1	1	1	0	1	1	0	1	0
*M*10	1	1	1	0	0	1	0	1	0	0

Table 3
Parameter setting model

	Example 1		Example 2		
	Cell I	Cell II	Cell I	Cell II	Cell III

L_k	2	2	2	2	2
U_k	4	4	6	6	6
LP	2	2	2	2	2
Min_Ut_k	0.5	0.5	0.7	0.7	0.7

Table 4a
ART1 solution without reversing the zeros and ones

		Part					
		2	5	6	1	3	4
Machine	1	1	0	1	0	0	1
	2	0	1	1	0	0	0
	3	1	1	1	0	0	0
	4	0	0	0	1	1	0
	5	0	0	0	1	1	0
	6	1	1	0	0	0	1

Table 4b
ART1 solution with reversing the zeros and ones

		Part					
		2	4	5	6	1	3
Machine	1	1	1	0	1	0	0
	2	0	0	1	1	0	0
	3	1	0	1	1	0	0
	6	1	1	1	0	0	0
	4	0	0	0	0	1	1
	5	0	0	0	0	1	1

3.1. Example 1 solutions

Results of a 6 × 6 machine-part matrix using ART1 without reversing the zeros and with reversing the ones are shown in Tables 4a and 4b, respectively, from the literature ([4]). Results from a binary system-based graph-neural network technique are shown in Table 5 from the literature ([7]).

Our suggested model, which is based on a newly-developed mathematical model that the Lingo package solver has optimally solved, yielded the findings shown in Table 6.

3.2. Example 2 solutions

The findings of the 10 ÷ 10 machine-part matrix, as acquired from the literature ([4]) using ART1, are included in Table 7.

Results from a binary system-based graph-neural network technique are shown in Table 8 from the literature ([7]).

The findings of our suggested model, which is based on a novel mathematical model that is optimally solved using the Lingo package solver, are included in Table 9.

Table 5

Graph-neural network solution

		Part					
		2	5	6	4	1	3
Machine	3	1	1	1	0	0	0
	1	1	0	1	1	0	0
	2	0	1	1	0	0	0
	6	1	1	0	1	0	0
	4	0	0	0	0	1	1
	5	0	0	0	0	1	1

Table 6
Proposed model solution

		Part					
		1	3	2	4	5	6
Machine	4	1	1	0	0	0	0
	5	1	1	0	0	0	0
	1	0	0	1	1	0	1
	2	0	0	0	0	1	1
	3	0	0	1	0	1	1
	6	0	0	1	1	1	0

Table 7
ART1 solution

		Part									
		1	2	6	3	7	8	4	5	9	10
Machine	1	1	1	1	0	0	0	0	0	0	0
	2	1	0	0	0	0	0	0	0	0	0
	3	1	0	1	0	0	0	0	0	0	0
	4	0	0	0	1	1	1	0	0	0	0
	5	0	0	0	1	0	1	0	0	0	0
	10	1	1	1	1	0	1	0	0	0	0
	6	0	0	0	0	0	0	1	0	1	0
	7	0	0	0	0	0	0	1	1	1	1
	8	0	0	0	0	0	0	1	1	1	1
	9	1	1	1	1	1	0	1	0	1	0

Table 8
Graph-neural network solution

		Part									
		1	2	6	7	4	5	9	10	3	8
Machine	9	1	1	1	1	1	0	1	0	1	0
	10	1	1	1	0	0	0	0	0	1	1
	1	1	1	1	0	0	0	0	0	0	0
	3	1	0	1	0	0	0	0	0	0	0
	2	1	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	1	1	1	1	0	0
	8	0	0	0	0	1	1	1	1	0	0
	6	0	0	0	0	1	0	1	0	0	0
	4	0	0	0	1	0	0	0	0	1	1
	5	0	0	0	0	0	0	0	0	1	1

4. Discussion

The traditional algorithms in the design of cellular manufacturing systems are not reliable in the case of large scale problems and are also unreliable in clear cell formation in the presence of bottleneck machines/ or bottleneck parts. In the cell formation problem, minimizing the exceptional elements is crucial criteria,

Table 9
Proposed model solution

		Part									
		4	5	9	10	1	2	6	3	7	8
Machine	6	1	0	1	0	0	0	0	0	0	0
	7	1	1	1	1	0	0	0	0	0	0
	8	1	1	1	1	0	0	0	0	0	0
	1	0	0	0	0	1	1	1	0	0	0
	2	0	0	0	0	1	0	0	0	0	0
	3	0	0	0	0	1	0	1	0	0	0
	9	1	0	1	0	1	1	1	1	1	0
	10	0	0	0	0	1	1	1	1	0	1
	4	0	0	0	0	0	0	0	1	1	1
	5	0	0	0	0	0	0	0	1	0	1

as well as the machine/or cell utilization. In the present approach, we could obtain the optimal solution of cell formation with considering the above criteria simultaneously.

If we consider the results of example 1 and comparing with Chen and Cheng [4] and Mahdavi et al. [7], we would obtain the specific advantages of present model as shown in Tables 10 and 11.

Also, the comparative results for example 2 are given in Tables 12 and 13.

Table 10
The comparison of three algorithms (Example 1)

	Cell I	Cell II	Cell III	STD[a]
Cell utilization (%)				
ART1 solution without reversing the zeros and ones	77.77	100	100	12.83
ART1 solution with reversing the zeros and ones	62.50	100	–	26.52
Graph-neural network solution	68.75	100	–	22.09
Proposed model solution	68.75	100	–	22.09

[a] Standard deviation of cell utilizations.

Table 11
The comparison of three algorithms (Example 1)

	Exceptional elements	Voids
ART1 solution without reversing the zeros and ones	3	2
ART1 solution with reversing the zeros and ones	0	5
Graph-neural network solution	0	5
Proposed model solution	0	5

Table 12
The comparison of three algorithms (Example 2)

	Cell I	Cell II	Cell III	STD[a]
Cell utilization (%)				
ART1 solution	66.67	77.78	75.00	5.78
Graph-neural network solution	65.00	83.33	100.00	17.51
Proposed model solution	83.33	80.00	83.33	3.70

[a] Standard deviation of cell utilizations.

Table 13
The comparison of three algorithms (Example 2)

	Exceptional elements	Voids
ART1 solution	8	9
Graph-neural network solution	6	9
Proposed model solution	6	6

When faced with ties for allocating parts to cells, such parts 3 and 7, we can gain better choice using the current approach compared to Mahdavi et al. [7]. If this is the case, we may use improved machine/cell utilization to allocate the appropriate components to cells.
Lastly, the current model demonstrates that the ideal outcome for generating a superior block-diagonal of cell formation has been achieved by maximizing the amount of voids and exceptional elements.

5. Conclusions

To achieve a greater degree of cell and block diagonal similarity in cell formation issues, it is necessary to minimize the amount of unusual parts and voids.
By comparing the suggested model to two established methods in the literature, we can see that the current model outperforms the others from the perspective of cell utilizations.

References

[1] "Mathematical model for cell formation considering investment and operational costs," published in the European Journal of operating Research, volume 69, issue 3, 1993, pages 330–341, by G.K. Adil, D. Rajamani, and D.A. Strong.
One mathematical method for the construction of production cells was proposed by Z. Albadawi, H.A. Bashir, and M. Chen in 2005 in the journal Computers and Industrial Engineering, volume 48, issue 1, pages 3-21.
In Production Engineer 42 (1963) 472, J.L. Buridge discusses production flow analysis.
“A neural network-based cell formation algorithm in cellular manufacturing” published in the International Journal of Production Research in 1995, pages 293–318, was written by S.J. Chen and C.S. Cheng.
In 2006, F.M. Defersha and M. Chen published an article in the International Journal of Production Economics titled "A comprehensive mathematical model for the design of cellular manufacturing systems." The article can be found on pages 767–783.
The authors of the article "A trade-off analysis between process plan selection and cell formation in cellular manufacturing systems" (23 (7-8) (2004) 501-506) conducted research in the field of advanced manufacturing technology.
International Journal of Production Research 39 (13) (2001) 2913-2922, by I. Mahdavi, O.P. Kaushal, and M. Chandra, a graph-neural network method to cellular manufacturing based on a binary system.
D. Rajamani, N. Singh, and Y.P. Aneja published an article in the European Journal of Operational Research in 1992 titled "Selection of parts and machines for cellularization: a mathematical programming approach" (62, 47–54).
[9] In Robotics and Computer-Integrated Manufacturing, J. Slomp, B.V. Chowdary, and N.C. Suresh (2005) used a mathematical programming technique to design virtual manufacturing cells, which they published in volume 21, issue 3, pages 273-288.
Computers and Industrial Engineering 34 (4) (1998) 759-776, by S.M. Taboun, N.S. Merchawi, and T. Ulger. A two-stage approach for cost-effective part family and machine cell construction.
[11] The authors of the paper "Design of a facility layout problem in cellular manufacturing systems with stochastic demands" (Applied Mathematics and Computation 184 (2) 2007 721-728) are R. Tavakkoli-Moghaddam, N. Javadian, B. Javadi, and N. Safaei.
According to J. Wang's work published in Automatica 39 (9) (2003) 1607-1615, a linear assignment method was used to form machine cells and part families in cellular manufacturing systems.