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The Phases of Combining the Carriage of Goods for Indefinite, Fuzzy, and Stochastic Parameters in Mathematical Modeling

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Abstract: Uncertain hazards are a defining feature of combined cargo transportation in Ukraine. The article's goal was to provide a mathematical model for selecting the mode of transportation that, in the presence of fuzzy, stochastic, and unknown risk characteristics, would correspond to the best value of the integral objective function. The mathematical model's efficiency made it possible to create short-term forecasts in real time in addition to long-term forecasts that take a lot of time. This makes it possible to swiftly alter transportation conditions and routes. The assimilating character of certain uncertain risks was discovered through practical testing of the mathematical model. The article presents the analysis's findings. When such a risk is recognized, all transportation circumstances drastically change. Long-term predictions enable the prediction of new transportation conditions and routes.

Keywords: Mixed transportation, mathematical model, uncertainty, fuzzy and stochastic parameters.

1. Introduction

Combined transportation (CT) management is so complex in the presence of fuzzy elements and parameters, which are marked by a high degree of uncertainty, that the application of mathematical modeling techniques is frequently restricted to the processing of expert judgments. This causes estimates to be highly subjective, their relevancy to decline, and the cost of transportation to rise as a result. Because the CT operators will account for the impact of uncompensated risks in the cost of services, transportation will be more expensive under these circumstances. Numerous reasons contribute to the uncertainties surrounding multimodal transportation in Ukraine. These factors, in our opinion, include, among other things, the potential for widespread aggression, high national currency volatility, a considerable degree of undervaluation of the national currency, which can result in a significant increase in exchange rates due to panic in the foreign exchange market, etc.

The behavior of the system under the uncertainty factor was predicted using a variety of methods. Specifically, the system's state and numerical value of the output parameter that the disturbing signal operates on were simulated mathematically. P. Tomey, R. Ortega, and others have examined this mathematical model [1]. Adaptive techniques have started to be applied to such systems.

The idea of modeling systems using nonlinear generators was examined in order to assess the effects of imprecise and uncertain parameters. The works of M. Zapateiro, Y. Vidal, and L. Acho [1] examined control of systems in the face of chaotic external stimuli. In the works of L. Marconi, L. Praly, R. Marino, G.L. Santosuosso, P. Tomei, and others, control of the system by compensating for nonlinear external influences was studied under the condition of the generalization of the system model [2], [3], [4]. [5], [6]. The intricacy or unsuitability of using these scientific works for assessing and accounting for the uncertainty elements in CT modeling has been assessed. However, the proposed algorithms made use of some mathematical methods for resolving the issues in the aforementioned publications. Graph theory is one of several methods used for mathematical modeling of transport systems [7, 8, 9, 10, 11]. We believe that the method used in the work [12] for mathematical modeling of mixed (multimodal) transportations is better suitable. The work of controlling the CT should, in our opinion, be viewed as selecting the optimal path for the chosen goal function from the starting point to the destination point in real time, replacing some stages if they do not correspond to the target function. By definition of CT, the selection of path steps is carried out in line with the optimization of the chosen integral goal transport function and are guaranteed to be available at the time of the problem. There is a lot of ambiguity involved in selecting the optimum path option from the alternatives at the beginning and at each level. Issues that come up when attempting to solve this problem:

1. The mathematical techniques employed at each level are not comparable.
2. The existence of uncertainty in the CT process's a priori adaptation method selection.
3. The uncertainty associated with the effectiveness function's non-monotonicity during cargo transportation.
4. Algorithmic instability. It has to do with how uncertain the integral target function's parameters are.
5. The creation of a universal mathematical model for all transportation stage variations and target function parameter

sets, with the selection of the most effective algorithm based on the uncertainty factors for each CT stage. This article's goal was to provide a mathematical model for choosing the transportation route that would match the optimal value of the integral target function, with the option to modify the route at each stage of transportation in accordance with uncertainty factors as well as fuzzy and stochastic parameters.

2. Methodology

In the work [12], the problem of multimodal (combined) transportation of cargoes is proposed to be considered by means of the construction of an appropriate digraph. The factor of uncertainty and fuzzy variables complicates the implementation of the method described in [12].

To solve this problem, we propose to use the method of finding the best path that corresponds to the integral destination function of transportation, in a fuzzy orthography, in an adaptive manner, using soft calculations, minimal dominant half-variants of sets of nodes or edges of a graph. By definition, a semi-variant, unlike an invariant, can be modified in some way.

Uncertainty leads to the need to generalize the method described in [12], using the recommendations given in [14], [15] applying intuitionistic dominant half-variants (IDNs), unlike fuzzy sets, IDNs are characterized to some extent by independence of degrees of membership and non-membership.

That is, the IDN defined on the continuum $\boldsymbol{W}$ can be represented as:

$$Q = \{\langle w, \mu_Q(w), \vartheta_Q(w)\rangle | w \in W\} \quad (1)$$

where $\mu_Q(w) \in [0,1]$ та $\vartheta_Q(w) \in [0,1]$ - accordingly are the degree of membership and non-membership of $\boldsymbol{w}$ to $\boldsymbol{Q}$ provided

$$(\forall w \in Q)[(\mu Q + \vartheta Q) \leq 1] \quad (2)$$

Then the digraph for the intuitionistic fuzzy set (IFS) when the edges and nodes belong to the intuitionistic set can be represented as $P = (Q, \boldsymbol{H})$ where $Q = \langle B, \mu_Q, \vartheta_Q\rangle$ B - the corresponding IFS is defined at the set of nodes **B,** $\boldsymbol{H} = \langle B \times B, \mu_H, \vartheta_H\rangle$ IFS for which

$$\mu_H(wl) \leq min\,(\mu_Q(w), \mu_Q(l)) \quad (3)$$

$$\vartheta_H(wl) \leq max\,(\vartheta_Q(w), \vartheta_Q(l)) \quad (4)$$

subject to the condition

$$(\forall w, l \in B)[0 \leq (\mu_H(wl) + \vartheta_H(wl)) \leq 1] \quad (5)$$

If we denote the degree of dominance by $\alpha(W)$, , then the dominant IFS W_α which, from the point of view of control theory, will represent the so-called "trunk set" belonging to the continuum $\boldsymbol{W}$ for the digraph P and will determine the

"trunk graph" as a term. The route that corresponds to the integral target function's optimum or, if required, suboptimal can be found using this trunk graph. As a suboptimal of the integral target function, we realize threats that were previously specified as uncertainties for a certain route in order to find the ideal value of this function for the route variations on the digraph P."
When a bounded set of nodes and edges is part of the so-called "navigational" set of nodes and edges of a continuum W for a digraph P^1, it is referred to as a "trunk" since it allows for the realization of a version of a route from the initial to the end node of this route. In other words, the so-called "navigation" graph includes a graph with a predetermined, limited collection of nodes and arcs (the so-called trunk graph).

3. Algorithm, results and discussion

The suggested mathematical model belongs to the category of "rough" (or "hard") pseudo-dynamic models. The usage of digraph restricts the system's direct dependence on time, which is why it is classified as pseudo-dynamic. In this instance, the implicit dependence on time is only evident in a series of static states of the model that correspond to the real-time flow. The fact that the suggested model assesses the system's limits of instability when minor parameter changes result in a change in the so-called number of degrees of freedom explains why it is categorized as "not rough". This makes it possible to create an algorithm (see Fig. 1) that will require a lot of computer power and take longer to compute, but will yield a more pertinent outcome and, to a larger extent, satisfy practitioners' needs. This is because fuzzy sets of much higher order are used when a collection of nodes and edges are members of the IFS.

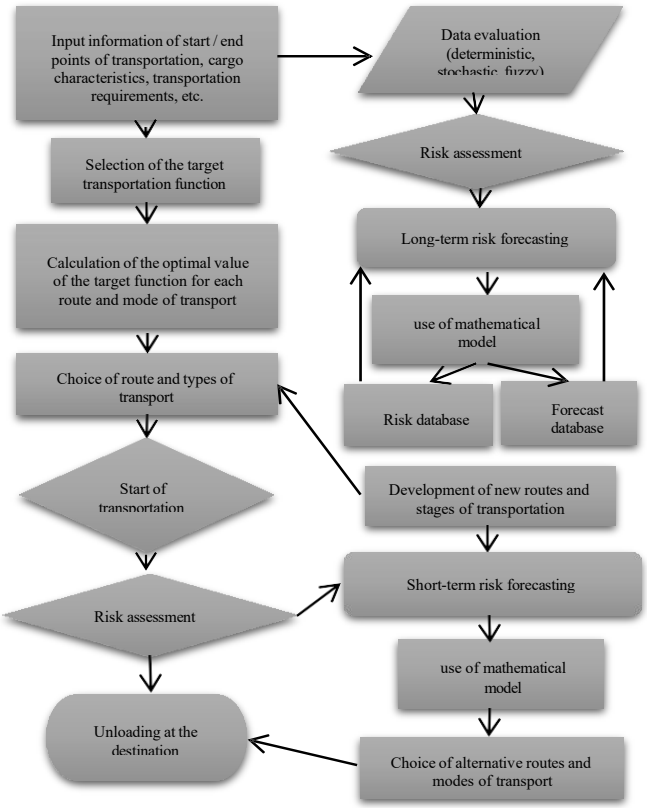


Fig. 1 - Flowchart of algorithm implements a universal mathematical model

Then the algorithm of using IFS will have the following steps:

- formation of IFS as a collection of all nodes and edges that can be assigned to the navigation set;
- charting of the navigation graph based on the existing navigation sets;
- search for subsets in this graph that can be defined as the minimal dominant ones;
- charting of a trunk graph, which includes the found minimal dominant subsets;
- transformation by way of adaptive soft calculations of indeterminate variables to fuzzy or stochastic ones;
- transition to the use of mathematical apparatus of regression-correlation analysis and fuzzy logic.

As shown in Fig. 1. The mathematical model is a necessary, effective, important part of the overall algorithm, but only a part of it. The result of using the model is creating a short-term or long-term forecast, using which logistics operators, owners of transport companies will decide on the feasibility of using any route of the MT or its particular sector, or any mode of transport within the changes in risk parameters. Using the mathematical model for a rapid change of the route will be possible only if the time required for forecasting is minimized.

The use of the apparatus of correlation-regression analysis in the case of multimetric regression (and the factors of CT, characterized by the presence of uncertainty, as the study showed, are multimetric) is complicated by the multicollinearity of the parameters. For cases where the mathematical apparatus of regression-correlation analysis cannot be used, it is suggested to use the mathematical apparatus of fuzzy sets, if necessary, combining them according to the algorithm [12].

Multicollinearity, by definition, is described by linear functions of interdependent parameters. As the evident functional relationship of the parameters for the CT is practically eliminated, it is worth focusing on the latent stochastic dependence of these parameters. To determine the communication density, we will apply an algorithm developed to perform this task. This algorithm is executed in the following sequence. The matrix of correlation coefficients of parameters is evaluated in pairs. The corresponding multimetric correlation and determination coefficients are defined. Given a certain level of significance of these coefficients, the formation of the regression equation is performed The check of the functional dependency of the specified parameters is envisaged. If confirmed, this indicates complete multicollinearity. Then the parameter matrix will be degenerate, which by definition means the absence of the inverse matrix. Namely, the inverse matrix is required for the least squares estimation. That is, under this condition, we cannot find the regression equation. In this case, we use soft calculations. But complete multicollinearity is unlikely. Partial multicollinearity leads to an increase in the variance of factor variables, which leads to a decrease in the accuracy of interval estimates, the formation of incorrect estimates of the significance of the parameters, the inability to determine the individual influence of the parameter on the relevant factor, the instability of estimates using the least squares method. An indicator of the presence of multicollinearity is the approximation of the determinant of the correlation matrix to zero. The approximation of the determinant of the correlation matrix to one will indicate the absence of multicollinearity. To reduce multicollinearity, we use adaptive algorithms to sequentially exclude parameters or transform variables by applying not directly the parameters, but the differences of their values at the beginning and end of each stage of transportation. The next step in the algorithm is to check the reduction degree of the determination coefficient. This, in turn, allows us to estimate the trends of increasing or decreasing the relevance of the mathematical model.

As a result, for the multicollinear factor θ, which depends on the parameters z_j an analog of the known equation is used

$$\vec{\theta} = a + \vec{\vec{b}} * \vec{z} \quad (6)$$

where θ is the vector of risks, $\vec{\vec{b}}$ is the vector of regression equation coefficients, $\vec{z}$ is the vector of parameters.

Or, if necessary, the above form of equation (1) is used to determine the resultant feature.

If it is necessary to determine the change of parameters in time, the following dependencies are used

$$dz/d\tau = \gamma(\vec{z}, \vec{v}) + f(\vec{z}, \vec{v})\vec{\mu} \quad (7)$$
$$g = \delta(\vec{z}, \vec{v}) \quad (8)$$
$$\vec{v} = M(u, \vec{v}) \quad (9)$$

where $\vec{z}(\tau) \in R^n$ is a vector that determines the state of the n-dimensional system at time τ; $\vec{\mu}(\tau) \in R^k$ is a vector that defines a set of $\boldsymbol{k}$ control actions at time τ; $\tau \in (\tau_i \ldots \tau_{i+1})$, τ_i is the time of the beginning of the i-th stage of cargo transportation; τ_{i+1} is the time of the end of the i-th stage of cargo transportation; $\vec{g}(\tau) \in R^k$ is a vector that determines the limits of the intervals of admissible parameter changes at timeτ; $\vec{v}(\tau) \in R$ is the vector of the state of external influence on the system; $u \in R$ is the vector of uncertain parameters, δ, γ, f, M are the vector functions of the specified variables.

We have implemented the CT management algorithm in such a way as to ensure coordination of two tasks: efficiency of the solution and minimization of the cost of transportation under the conditions:

$$\tau_i \in [0, \tau], \quad (10)$$
$$C_i \leq C_{max}, w_j \in W, \boldsymbol{i} \in \{0, i_{max}\} \quad (11)$$

where τ_i – is the time of the CT stage, C_i is the cost of the transportation stage.

The algorithm for this part of the problem can be formulated as follows. We divide the continuum $\boldsymbol{W}$ into sub continuums of stochastic parameters and fuzzy variables.

Then for the sub continuum of stochastic parameters:

$$T = \{(\vec{w^{\rightarrow}}, \vec{p}) \langle F, \circledcirc \rangle \beta_F \} \quad (12)$$

where $(\vec{w^{\rightarrow}}, \vec{p})$ is the part of the continuum W for which each component of the vector $\vec{w^{\rightarrow}}$ is stochastic. That is, the component of the vector $\vec{w^{\rightarrow}}$ corresponds to a component of the probability vector $\vec{p}$, which can be described by an algebraic system with an integral target function F, that has all the properties of the so-called "compositions" $\circledcirc$, including seclusion, associativity and, all other things being equal, cargo transportation, idempotency.

Then the conditions of equation (7) can be represented as

$$\tau = \sum_0^i \tau_i \quad (13)$$

$$\mathbf{C}=C_1 * C_2 * \ldots * C_i \sum(+/-\Delta_k) \rightarrow min \quad (14)$$

where the sign $*$ denotes the effect of adding on the additive cost function, or the multiplication action on its multiplicity, Δ means the total possible profits (such as bonuses) or losses (eg penalties).

For the sub continuum of fuzzy variables to which ***FN***-number arithmetic (Fuzzy Naturale) is applied, we can write:

$$T = \{(\vec{w^{\rightarrow}}/\vec{N^{\rightarrow}}) \langle F, \circledcirc \rangle \beta_F \} \quad (15)$$

where $\vec{N^{\rightarrow}}$ is the vector of relations.

The uncertainty of the parameter is reduced to the inaccuracy of the use of soft calculations in the modeling of a particular stage of transportation, characterized by the specified parameter according to the method described in [15]. This is done by replacing the indeterminate parameter (tensor) with its invariants using the Kronecker method to form the product "variable - function of membership". That is, summing up, the reduction of an indefinite parameter to a Kronecker product of fuzzy variables.

Using graph theory methods to formulate a route, this will be realised on a digraph by the algorithm of selecting nodes closest to the current one which are connected with the terminal route by intermediate edges and nodes with no signs of uncertainty. Then the problem is reduced to the method described in [12] for finding the optimal path l_{opt} by selecting more efficient nodes (edges, stages of transportation) on the digraph.

$$l_{opt}(w_0 \circledcirc \{w_j\}) = min_{C(w_0 \circledcirc \{w_j\})} \sum l_j \quad (16)$$

under the condition that:

$$C \rightarrow C_{opt}(w_0 \circledcirc \{w_j\}) \quad (17)$$

where l_j, $\vec{w^{\rightarrow}}_j$ - $j-$ i route stages and parameter vectors defining these stages.

The above algorithm implements a universal mathematical model (see Fig. 1), which is suitable not only for management tasks but also for predicting the CT.

With the use of this model, the gross volume of combined transportations of Ukraine by individual types of cargo for the years of 2020 and 2021 was carried out with uncertain parameters.

The result of forecasting using the work data [16, 17] is shown in Fig. 2, Fig. 3, Fig. 4.

The prediction is made in optimistic and pessimistic versions. The optimistic variant is calculated based on the set of existing risks commonly found in CT, by changing the parameters of the statistical and fuzzy parameters of their impact within the limits of the forecast estimates. The pessimistic variant is calculated by implementing the two worst case scenarios of undetermined risks: intensification of military actions and domino effect in the national currency exchange rate. For the variant of the fall in the national currency exchange rate, we used the scenario of 2016, when within a few weeks there was an almost 2.3 times fall in the national currency exchange rate. The optimistic forecast in the figures is indicated by a solid line, pessimistic (military actions) is shown by a dotted line, pessimistic (change in the national currency exchange rate) is shown by a chain-dotted line. The data of Seaports Administration of Ukraine were used for the analysis

To forecast the salary for 2020 and 2021, calculations were made, taking into account two assimilating risks - the risk of large-scale military actions and the risk of the drop in the value of the national currency. Due to the assimilative nature of each of the above risks, forecasting was performed separately for each of them. The forecasting results are shown in Table 1. The use of mathematical modeling methods revealed a non-obvious result. It is estimated that the impact on the volume of transportation from the risk of the national currency drop will be greater than the impact from military actions.

Table 1 - CT forecast data for 2020 and 2021

Types of cargo	Forecast for 2020, thousand tons		Forecast for 2021, thousand tons	
	Optimistic	Pessimistic	Optimistic	Pessimistic

		hostilities	currency exchange rate drop		hostilities	currency exchange rate drop
Containers	1190350	510200	920200	1351500	There is no	712200
Food loads	124	50	71	79	There is no forecast	51
Bulk cargo, except cereals	15200	9580	14200	16582	There is no	10900

Regarding the impact of the cargo type. Reduction of the volume of containerized cargo transportation in 2020 in the presence of assimilating risks in relation to the optimistic option by 21% -57.1%, in 2021 - by 47%. For food cargoes, respectively, by 42.7% - 58.7%, and by 35.5%. For bulk cargoes, respectively, by 6.6% - 39%, and by 34%. Thus, it is proved that the impact of assimilating risks will also depend on the cargo. And the most vulnerable will be container and food transportation, the least - bulk cargo.

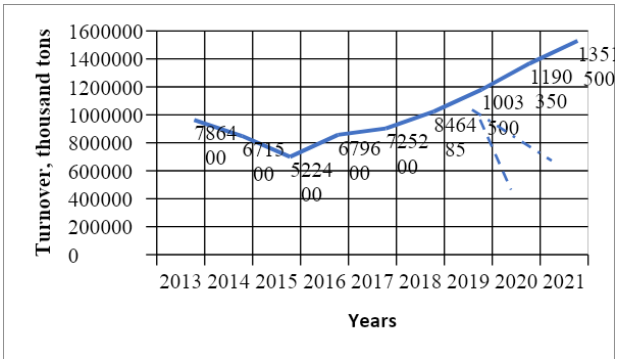


Fig. 2 - Container transportation volume forecast for 2020 and 2021

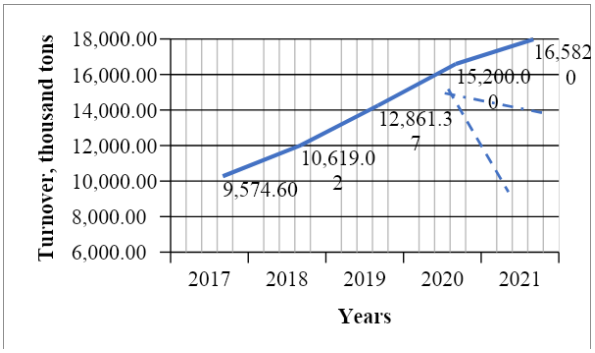


Fig. 3 - Bulk freight traffic forecast (excluding cereals) for 2020 and 2021

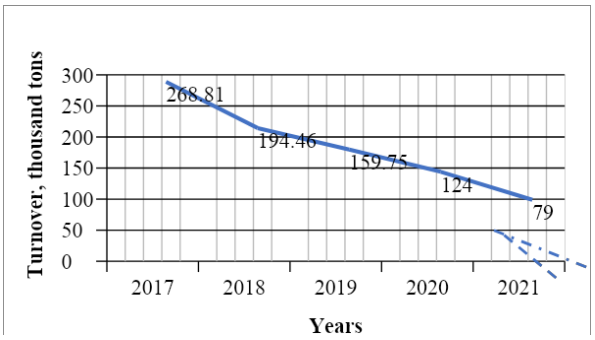


Fig. 4 - Estimation of the volume of transportations of food cargoes for 2020 and 2021

The forecast was made for the three types of cargo: container transportation, food cargo, bulk cargo with the exception of grain. It is these types of goods that correspond to the definition of combined transportations. These types of cargo have been subjected to pooled analysis in order to clearly illustrate the possible multifaceted effects of parameters specific to definite commodity markets.

In the optimistic variant of combined transportations, the tendency to increase their volume, more or less, depending on the types of cargoes, will be sustained. In the pessimistic scenario, shipment is forecast to decline to almost the level of 2015.

This confirms the argument that a definite number of the indeterminate parameters form the so-called “absorbing" risks whose impact on the result is crucial. In the case of absorbing risk, the overall impact of other risks will be negligible in comparison with it.

4. Conclusion

For the management of the CT and forecasting its outcomes, a universal algorithmic mathematical model is created that takes into account the impact of both fuzzy and stochastic parameters as well as indeterminate values on transit efficiency. The examined example of projecting the volume of combined transportation in Ukraine for future periods has demonstrated that uncertain parameters might provide absorbing risks, whose impact on the outcome may be bigger than the impact of the entire collection of other CT risks. Determining the transportation cost, estimating volume, and choosing CT routes can all be impacted by anticipating such hazards. The forecasting method is altered by the application of the suggested mathematical model.

The computation time and required computer resources have been greatly decreased by the suggested way of applying the proper mathematical tools to discover the optimal path that corresponds to the chosen objective function. Before risk factors hinder transportation, long-term projections enable the development of new routes and the testing of alternative modes of transportation for specific cargoes.

The mathematical model's pilot testing have shown that it can be utilized for both short-term forecasts in real time and long-term projections, which take some time. This demonstrated that logistics firms may use a mathematical model to swiftly and optimally alter shipment routes.

Logistics businesses are already using the projections of container transportation volumes, bulk cargoes (apart from grain), and food cargoes for 2020 and 2021 that were created using mathematical models.

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