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Giving pupils a contextualized understanding of differential equations using mathematical modeling

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ABSTRACT

In many fields, including mathematics, biology, computer science, and engineering, using mathematical modeling to get practical insight into real-world issues is highly regarded. One useful technique for modeling is differential equations. Students can interact with differential equations in a contextual setting through modeling. Teaching mathematics in context has the potential to provide students with a foundation for their understanding of the subject and serve as cognitive roots. In light of this, how can instructors employ mathematical modeling to give students contextual differential equation learning experiences? Three case studies were used in a study conducted in New Zealand. A mathematical modeling course, a lecturer, and students participated in each case study. All of the course material included differential equation-based modeling exercises and examples. This essay will include details about each course, the primary modeling exercises that each course employed, and instances of how students applied differential equations to these exercises. Additionally, information on the three speakers' shared approaches to teaching differential equations through mathematical modeling will be shared.

KEYWORDS

Mathematical modelling; Differential equations; Teaching approaches

1. Introduction

Three case studies were used in a study conducted in New Zealand. Every case study included students, lecturers, and participants in a mathematical modeling course. Participants' signed consent forms and participant information sheets were used to acquire their informed consent. Every lecturer taught modeling in a different way. All of the course material included differential equation-based modeling exercises and examples. This essay will provide details about each subject, the primary modeling exercises utilized in each course, and instances of how students applied differential equations to these exercises. Differential equations and other modeling techniques were included in the course materials, albeit each course employed a distinct set of modeling exercises. The modeling approach was emphasized differently by each presenter. Every modeling exercise was conducted in groups of students.

Table 1. Summary of modelling activities for each course.

Course	Modelling activity used	Focus on the modelling cycle	Students' responsibility for decisions
Course one	Open-ended	Explicit	All decisions, including the foundation model to use
Course two	Case Studies	Implicit	Modification of model only
Course three	Popular Culture	Explicit	All decisions except for the foundation model

A summary of the type of modelling activity used, focus on the modelling cycle and student responsibility for decisions during the activity is given in Table 1.

2. Course one – open-ended modelling activities

In the first case study, the students participated in a group open-ended modeling exercise that accounted for 5% of their final score after the lecturer explained modeling concepts and the modeling process during lectures and

tutorials.

The lecturer prepared the students to participate in the modeling activity by teaching them modeling techniques, learning about the mathematical modeling process, developing skills for productive group and teamwork, and comprehending the evaluation criteria for the modeling project.

Lectures covered the development of modeling methodologies, which included mathematical methods, tools, and techniques. Techniques for differentiation, such as analytical, numerical, and integration methods, were taught. Additionally, the course material addressed probability, vectors, and matrices. According to the lecturer's interviews, the use of proportionality relationships, dimensional analysis, optimization, rates of change, physical laws like Newton's second law, and curve fitting were among the particular modeling methods discussed in the course content.

Throughout the course, students were exposed to a generic modeling method both explicitly and contextually, which helped them gain an understanding of the mathematical modeling process. The presenter demonstrated how the modeling techniques contribute to the modeling process, and the course was "embedded within the context of the modeling process" (Spooner, 2020, p. 484). With the first teaching module devoted to the modeling process, students were introduced to the modeling process at the beginning of the semester. The lecturer's role modeling of the modeling process in lectures and applications throughout the course, references to the modeling process in completed assignment solutions, and the modeling process described in assessment instructions provided additional exposure to the modeling process. The general modeling process involved identifying the problem, which included defining the problem, enumerating and categorizing the factors, making assumptions, and creating a precise problem statement; creating a mathematical model, which involved using mathematical symbols to represent the factors and translating the assumptions into mathematical equations and inequalities; solving the model, which involved using analytical and numerical techniques to obtain a solution; interpreting the mathematical solution, which involved comparing the solution with reality. "When role modeling the modeling problems and in assessment solutions, the lecturer used the generic modeling cycle and the "keep it simple" principle." (Page 586 of Spooner, 2021).

During a pre-modelling project instruction, strategies for productive group/teamwork were learned. Activities for students to get to know one another and advice on how to collaborate well in groups were included in the pre-modelling session.

The pre-modelling project lesson also included a summary of the modelling process and an understanding of the criteria for the modelling project's assessment. The project instructions also covered time management, reference materials, report writing, required and permitted resources, and a project template for the group's final report.

The modeling exercise for course one involved modeling an open-ended, poorly specified situation using pertinent mathematics and physics. The problem was subject to interpretation because it was poorly stated. Students were able to create a mathematical model for the problem by "engaging in, and independently developing, a group-directed modelling process" (Spooner, 2021, p. 585). Questions like "How high can you jump from a building without causing injury" are used in different semesters. How large would the resulting crater be if a Cube-Sat, a tiny satellite with a square shape, struck the moon? In pre-arranged groups of four, students spent seven hours working on the question from their semester. The speaker chose the groups to guarantee a diversity of skills and genders.

How large would the ensuing crater be if a CubeSat struck the moon? did not use differential equations to generate student models, although some students employed these methods when modeling How far can you leap from a structure without being hurt? For instance, Steph's group first created a model that represented the downward and upward forces applied to the bones from the jump as being equal in order for the bone to break. This model was based on Newton's laws, particularly Newton's second law and the rules for kinetic and potential energy. However, they gave up on this model after realizing that time (t) would be required to determine the fall's height (x). According to their written report, the group's original model—which was subsequently abandoned—was as follows:

$$mg - cv^2 = -m\frac{d^2x}{dt^2}$$

$$\frac{mg - cv^2}{-m} = \frac{d^2x}{dt^2},$$

$$\frac{dx}{dt} = \int \frac{mg - cv^2}{-m},$$

$$x = \int \left(\frac{mg - cv^2}{-m}\right) dt.$$

The possibilities of an open-ended modelling activity are that they allow a student to creatively adapt a differential equation to the situation by applying a modelling process. This allows students to experience first-hand connecting calculus to real-world applica-tions. The creative aspect and open-ended nature of the modelling allow students to be fully engaged in the process. Full engagement in the process generated student benefits of retention and adaptivity of the skills gained through their experience. This leaves students with a first-hand application of differential equations to model a real-world situation. This, in turn, should help develop students' confidence, or self-belief, in themselves as users of mathematics.

3. Course two – case studies

For the second case study, modelling techniques, including mathematical tools, were taught during lectures and tutorials. The students experienced the modelling process by studying modelling case studies. Presenting the case studies to the class contributed to 10% of their final grade.

In preparation for the case studies the topics taught included: discrete equations, ordi-nary differential equations in general, models of population dynamics using ordinary differential equations, recap on solving differential equations and the difference between difference equations and differential equations.

Students were presented with three case studies. The three cases were: *Case Study: Prickly pear meets its moth* (Barnes & Fulford, 2015); *Case Study: Possums threaten New Zealand cows* (Barnes & Fulford, 2015) and *Case Study: It's hot and study in the attic* (Barnes & Fulford, 2015). Students were asked to read the case studies and then rank the case studies from 1 to 3 according to which case study they preferred to work on. Using the students' ranking, the lecturer then matched students with case studies ensuring that all student groups were doing a different case study. Three student groups were generated with 3, 2 and 2 students.

The modelling activity consisted of groups preparing and giving 3 separate presenta-tions, based on their case study, to the class. The first presentation involved explaining the background of the model, presenting the model and defining the variables of the model. The second presentation involved presenting the analysis of the model, including equilib-rium points, phase portraits and graphs. The third presentation involved identifying the limitations of the model and possible modifications to the model, followed by working through a modification to the model, including presenting the analysis of the modifica-tion and its implications. At the end of each presentation, the audience was expected to ask questions, with assessment marks given for quality questions asked.

Each time students presented was a teaching and learning opportunity. The content pre-sented in the presentation was used as teaching points by the lecturer for the class. For example:

The Ngaire and Natalia group used a method not covered in lectures for finding the general behaviour and stability of one of the equilibrium points in the presentation of the analysis of the *Possums threaten New Zealand cows* model. Ngaire and Natalia first pre-sented the model including defining the variables and parameters, followed by how to find the equilibrium points and then presented how to find the eigenvalues of the Jaco-bian matrix. Presented below is a summary of the mathematics without a detailed student explanation:

As in the case study (Barnes & Fulford, 2015), the model is given as

$$\frac{dS}{dt} = b(S+I) - aS - \beta SI,$$

$$\frac{dI}{dt} = \beta SI - (\alpha + \alpha)I,$$

where

S = possums susceptible

I = possums infected (assumed to be also contagious)

b = natural birth rate

a = natural death rate

α_d = infection transmission coefficient (density dependent)

β = death rate from the disease.

The equilibrium points for the model can be established as follows:

$$(S, I) = (0, 0),$$

$$(S, I) = \left(\frac{\alpha_d + a}{\beta}, \frac{(b-a)(\alpha_d + a)}{\beta(\alpha_d + a - b)}\right).$$

To determine the stability and behaviour of the equilibrium points, the eigenvalues of the Jacobian matrix for the first equilibrium point can be found as $\lambda_1 = (b - a)$ and $\lambda_2 = -(\alpha_d + a)$.

For the second equilibrium, the algebra is not so straightforward. This is where Ngaire and Natalia, presented how, by examining the trace and determinant of the Jacobian matrix, the nature, stability and general information regarding the behaviour of the equilibrium points could be found without actually doing numerical calculations.

By examining the nature of the eigenvalues and the sign of the trace of the Jaco-bian matrix, the stability of the equilibrium point can be established. Ngaire and Natalia presented the following rationale to the class:

Case 1:

$$\text{Trace}^2(J) - 4\det(J) < 0$$

then eigenvalues will be complex numbers.

If Trace(J) < 0, then Re$(\lambda) < 0$. It is stable.

If Trace(J) > 0, then Re$(\lambda) > 0$. It is unstable.

Case 2:

$$\text{Trace}^2(J) - 4\det(J) > 0,$$

then the eigenvalues will be real numbers.

If det(J) > 0, then we have

$$\text{Trace}^2(J) - 4\det(J) < Trace(J)$$

Because $\text{Trace}^2(J) - 4\det(J) < \text{Trace}^2(J)$ then det(J) > 0.

- Trace(J) > 0, then $\lambda_1 > 0, \lambda_2 > 0$, so it is unstable.
- Trace(J) < 0, $\text{Trace}^2(J) - 4\det(J) < 0$, then it is a stable complex number.

If det (J) < 0, then we have

$$\text{Trace}^2(J) - 4\det(J) > Trace(J).$$

Because $\text{Trace}^2(J) - 4\det(J) > \text{Trace}^2(J)$

$det(J) < 0$.

$\lambda_1 > 0, \lambda_2 < 0,$

So, it is a saddle point, then it is unstable.

By examining the Jacobian matrix $\left(\frac{\alpha_d + a}{\beta}, \frac{(b-a)(\alpha_d + a)}{\beta(\alpha_d + a - b)}\right)$ the trace was determined to be

$$\text{Trace}(J) = \frac{(b-a)(\alpha_d + a)}{\beta(\alpha_d + a - b)}$$

Trace > 0 when $\alpha_d + a$ < b and equilibrium point is unstable. Trace < 0 when $\alpha_d + a$ > b and equilibrium point is stable.

By students needing to present the case study and the analysis of the model, it forced students to

engage with mathematics and explore ways of producing the analysis. Students giving presentations enabled the whole cohort to be exposed to content that had not been covered in class. Presenting the mathematics of the case studies allowed students to see the relevance of the mathematics and techniques, including the mathematics of, and tech-niques for, differential equations. Nicola illustrates how students found the presentation valuable commenting 'and I was like, that's cool that they covered it like. But yeah, it gives you, like, you never thought that it could be done that way sort of thing like it gave you an idea though. You could look at it this way'.

The lecturer used the student presentations to identify key teaching points relevant to the student's mathematical development for differential equations. In the case of teaching from Ngaire and Natalia's presentation, the lecturer focused the following lecture on the techniques the students had introduced to the class. This meant students could see the applicability and relevance of the mathematics as opposed to just being taught abstract skills. An abstract from the lesson observation is given below:

> For the 3rd fixed point [lecturer 2] introduced usinga solution to the general Jacobian to show you could tell a lot about the behaviour of a solution (phase plane) without actually having to numerically calculate it. You could look at the different signs of the terms in the solution to determine the behaviour of the solution.

General solution for eigenvalues of Jacobian $\lambda_{1,2} = \frac{Tr(J) \pm \sqrt{Tr^2(J) - 4det(J)}}{2}$.

By Ngaire and Natalia presenting their analysis to the class, they exposed their peers to a new technique for modelling, enabled the lecturer to pull out the new mathematics and use it as a teaching point as part of the feedback feedforward session of the students' pre-sentation. The students' presentation prompted the lecturer to follow this up by dedicating the next lesson to teaching this new technique in further detail. This created a situation for teaching as needed, showing students the relevance and applicability of mathematics.

Another example of using students' work as teaching points occurred when Nicola and Rhys's group presented a phase portrait to the class. After the presentation, the lecturer focused on the detail of phase portrait and used it as a teaching tool for behaviours of equilibrium points. In particular, the lecturer showed how the stability of the system is seen in a phase portrait. The teaching that occurred was described by Nicola as further developing our understanding of the 'stability and the different definitions of stability for that really oddly behaving fixed point' in their model. Again, the lecturer gave a follow-up lesson, this time on phase portraits.

As can be seen from these examples, using student presentations was a powerful way of teaching and enabled the lecturer to add to the students' knowledge by providing new mathematics and techniques relevant to what they were currently working on.

A benefit of the presentations was that the lecturer can use them as teaching points for the whole class. It was seen from the student interviews that in order for this to be success-ful, it is important to create an environment where students feel safe to make mistakes and have their mistakes and work in general, used as teaching points. A clear message com-municated to students by lecturer 2 was not to be worried if the mathematics was correct. We learn more from our own and each other's mistakes than getting things right all the time.

Different from Course 1, the modelling cycle was only briefly taught at the beginning of the course. It was the lecturer's intention that students would implicitly experience the modelling cycle by getting to know the case study models and draw on this in the modification stage of their work. This was successful for some students, but not all.

4. Course three – popular culture

For the final case study, modelling techniques were taught during lectures and tutorials. Modelling was experienced by creating models for different fictional scenarios.

To prepare students for the mathematical modelling activity, the students were first introduced to the modelling cycle with the lecturer developing simple models using known mathematics. This exposed students to what mathematical models were and how they could be simply developed. In two lessons before the modelling activity, the lecturer deliv-ered two lectures, primarily focused on the infectious disease S-I-R model, which students would use as the base for model development during the activity. The lectures were for students to get to know and be familiar with the S-I-R model and possible applications of the model. The lecturer role-modelled how to manipulate the model to fit different situations.

The mathematical modelling activity occurred in the lecture following the S-I-R lec-tures. This was a non-assessed modelling activity using fictional scenarios based around zombie movies and books. Students worked collaboratively to create a model to represent the movie or book situation assigned to their groups. The final fifteen minutes of the lecture were dedicated to student groups presenting what they had created. Student groups were selected by students receiving a make of sweets as they entered the lecturer theatre. All students who receiveda brandA sweet formed one group, brandB sweet formed another group, etc. Large groups were split as needed to achieve groups of 5 or 6 students each. As Leo explained in his interview '[lecturer 3] has pictures of all the wrappers and you go sit at the table with your wrapper.'

Groups were then assigned one fictional zombie activity to model. The activities designed by the lecturer were:

> Night of the Living Dead: A radioactive leak at a nearby military base leads to dead bod-ies becoming reanimated as zombies. The zombies attack and kill humans and can only be stopped by decapitation or by burning.
>
> 28 Days Later: A bacteria developed to cure cancer is released from a scientific lab after animal protestors try to rescue lab chimpanzees. The bacteria called 'rage', is spread by contact with blood. Those infected by 'rage' are turned into super-violent monsters in under a minute and try to attack humans. Those infected by 'rage survive until they die of hunger.
>
> I am a Legend: A bacterial disease has the ability to reanimate dead bodies, turning them into 'feral' vampires-like creatures which cannot tolerate sunlight, garlic, etc. However, it is discovered that if a live person gets infected, they do not become feral. Instead, they become a sentient 'vampire' race which has the potential to replace humans.
>
> Walking Dead: All humans carry a pathogen which activates when the host dies, turning them into a zombie which shambles after humans, attacking them when they can. The pathogen can only be destroyed by damaging the host's brain or by fully destroying the body (e.g. by cremation).
>
> Brain Dead: The bite of a Sumatran Rat Monkey infects the main character of a movie. The main character declines and degrades, before eventually dying and being reanimated as a violet zombie.

Other humans become zombies after they are killed. The only way to stop them is by fire and lawnmowers.

The students were given one lecture to collaboratively create a model to represent their given situation. The final ten minutes of the lecture were dedicated to student groups presenting what they had created.

Data were not collected on students' written work. However, students discussed in interviews how they went about developing their models. This is illustrated as follows:

Common for all student groups was to 'take the S-I-R model and change it' (Leo).

Hazel said 'we had to determine what were the important factors for our situation and incorporate them into the model'. Similarly, Leo and Luke both talked about 'identify-ing the variables and then assigning them to susceptible, infected or recovered' (Luke). Both Leo and Luke discussed how they then used their modified models to determine 'how the numbers of susceptible people and infected people and dead, and removed peo-ple changed over time' (Leo). Luke went on to say 'once we assigned the variables to the S-I-R model, we had to come up with another variable for the people that had been bit-ten but weren't infected or didn't die but they came back as a zombie' illustrating that modifying the S-I-R model for the situation was not straightforward and involved students actively having to think and apply themselves. Students enjoyed the modelling experience and got a lot out of the experience as Hannah illustrates when she said 'we got to apply modelling to, I wouldn't say a real situation, a situation and actually have a go at trying to model something. It made me feel smart. Ours was insanely complicated because we tried to incorporate all the variables, but we did alright. We ended up writing all the differential equations out and everything'. Hannah summarised the student experience of the activ-ity by saying 'we learned about the SIR model and how to manipulate it to fit together.' (Hannah).

Interestingly to note, students commented on how participating by watching the presen-tations allowed them to 'listen and see how others would approach it' (Luke) commenting on how they 'could learn that way as well' (Hope). This is similar to the course 2 students' experience who found watching other students present their work a valuable learning experience.

4.1. *Insights into common lecturer practices across all three courses*

All lecturers designed and delivered their instruction such that it included opportunities for lecturers to *role-model mathematical modelling behaviours, provide resources and pro-mote independence* for students. All lecturers created opportunities for students to work independently, while also promoting independence as a key modelling behaviour for students to develop.

4.2. *Conclusion*

Giving pupils the chance to apply differential equations while mathematically modeling helps them understand the value of mathematics and the variety of its applications. It is intended that by applying differential equations in contexts, students' comprehension of mathematics and its practical applications will be realized. Students learn how to use mathematics in unconventional ways and adapt it imaginatively to the particular situation they are studying. Students' model development has importance, even if it doesn't result in a finished, practical model. A rich contextual learning environment for differential equations and mathematics in general is offered by the use of modeling exercises. Students learn by participating in the activities on their own and by presenting and viewing each other's work.

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