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Properties of γ-πgγ-closed Functions That Are Associated

V.Kumar[1*], Parimalam[2]

[1]Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai District, Tamil Nadu, India.

[2]Department of Mathematics, CSI College of Arts and Science for Women, Madurai, Tamil Nadu, India.

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Abstract: [3, 7, 11] proposed the idea of γ-open sets. This paper's main goal is to use γ-open sets to introduce and explore pre-πgγ-closed functions.

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Introduction and Preliminaries

In general topology, generalized open sets are crucial, and many topologists across the globe are now studying them. The use of extended open sets to implement variably modified versions of continuity, separation axioms, etc., is, in fact, a major subject in both general topology and real analysis. The idea of γ-open sets, which was first proposed in [3, 7, 11], is among the most well-known concepts and a source of inspiration. Generalized closed sets in topological spaces were developed and examined by Levine [12] in 1970. Malghan [15] introduced generalized closed functions and derived certain regularity and normality preservation theorems in 1982. Generalized semi-open sets were introduced by Arya and Nour [5] in 1990, and they were then used to derive characterizations of s-normal spaces according to Maheshwari and Prasad [13]. Devi et al. [6] determined that the continuous generalized semi-closed surjective image of a normal space is s-normal in 1993 after defining and studying generalized semi-closed functions. The continuous generalized pre-closed surjective image of normal space is prenormal [17] (or p-normal [19]), as shown by Noiri et al. [16], who defined generalized preclosed sets and presented generalized pre-closed functions in 1998. Generalized β-closed functions have recently been described by Tahiliani [22], who also demonstrated that the continuous generalized β-closed surjective pictures of normal (or regular) spaces are β-normal [14] (or β-regular [2]). Additionally, it has been shown that continuous pre-β-open maintains β-regularity [14]. β-gβ-closed

Throughout this paper, X and Y refer always to topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A) and int(A) denote the closure of A and the interior of A in X, respectively. A subset A of X is said to be regular open [21] (resp. regular closed [21]) if A = int(cl(A)) (resp. A = cl(int(A))). The finite union of regular open sets is said to be π-open [23]. The complement of a π-open set is said to be π-closed [23]. A subset A of X is said to be β-open [1] (= semi pre-open [4]) if A ⊆ cl(int(cl(A))). A subset A of X is said to be γ-open [11] or sp-open [7] or b-open [3] if A ⊆ cl(int(A)) ∪ int(cl(A)). The complement of γ-open (resp. regular open) set is called γ-closed (= b-closed) (resp. regular closed). The intersection of all γ-closed sets of X containing A is called the γ-closure

[11] (= b-closure) of A and is denoted by γcl(A) (= bcl(A)). It is evident that a set A is γ-closed if and only if γcl(A) = A [18]. The γ-interior [11] of A, γint(A) or bint(A), is the union of all γ-open sets contained in A. A subset A of X is said to be γ-clopen [9] or b-regular [18] if it is γ-open and γ-closed.

The family of all γ-open (resp. γ-closed, γ-clopen, β-open, regular open, regular closed) sets of X is denoted by BO(X) or γO(X) (resp. BC(X) or γC(X), BR(X) or γCO(X), βO(X), RO(X), RC(X)). A subset A of a topological space (X, τ) is called a generalized γ-closed [8] (briefly gγ-closed) set of X if γcl(A) ⊆ U holds whenever A ⊆ U and U is open in X. A will be called gγ-open if X\A is gγ-closed. A subset A of a topological space (X, τ) is called $\pi g\gamma$-closed [20] set of X if γcl(A) ⊆ U holds whenever A ⊆ U and U is π-open in X. A will be called $\pi g\gamma$-open if X\A is $\pi g\gamma$-closed.

Lemma 1.1 ([20]). *A subset A of a space X is $\pi g\gamma$-open in X if and only if F ⊆ γint(A) whenever F ⊆ A and F is π-closed in X.*

Remark 1.2 ([11]). *Every open set is γ-open but not conversely.*

Remark 1.3 ([8]). *Every γ-open set is gγ-open but not conversely.*

Remark 1.4 ([20]). *Every gγ-open set is $\pi g\gamma$-open but not conversely.*

Remark 1.5 ([11]). *Every γ-open set is β-open but not conversely.*

Theorem 1.6 ([4]). *For any subset A of a topological space X, the following conditions are equivalent:*

A ∈ βO(X);

A ⊆ cl(int(cl(A)));

cl(A) ∈ RC(X).

Pre-$\pi g\gamma$-closed Functions

Definition 2.1. *A function f : X → Y is said to be pre-$\pi g\gamma$-closed (= γ-$\pi g\gamma$-closed) (resp. regular $\pi g\gamma$-closed, almost $\pi g\gamma$-closed) if for each F ∈ γC(X) (resp. F ∈ BR(X), F ∈ RC(X)), f(F) is $\pi g\gamma$-closed in Y.*

Definition 2.2. *A function f : X → Y is said to be $\pi g\gamma$-closed if for each closed set F of X, f(F) is $\pi g\gamma$-closed in Y.*

From the above definitions, we obtain the following diagram:

pre-$\pi g\gamma$-closed −→ regular $\pi g\gamma$-closed

↓ ↓

$\pi g\gamma$-closed −→ almost $\pi g\gamma$-closed

Remark 2.3. *None of all implications in the above diagram is reversible as the following examples show.*

Example 2.4. *Let X = Y = {a, b, c, d}, τ = {∅, X, {b}, {b, d}, {a, b, c}} and σ = {∅, Y, {c}, {d}, {b, c}, {c, d}, {a, c, d}, {b, c, d}}. Let f : (X, τ) → (Y, σ) be the identity function. Then f is both regular $\pi g\gamma$-closed and $\pi g\gamma$-closed but it is not pre-$\pi g\gamma$-closed.*

Example 2.5. *Let X = Y = {a, b, c, d}, τ = {∅, X, {a}, {a, d}, {a, b, d}, {a, c, d}} and σ = {∅, Y, {c}, {d}, {b, c}, {c, d}, {a, c, d}, {b, c, d}}. Let f : (X, τ) → (Y, σ) be the identity function. Then f is almost $\pi g\gamma$-closed but not $\pi g\gamma$-closed.*

Example 2.6. *Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is almost $\pi g\gamma$-closed but not regular*

$\pi g\gamma$-closed.

Lemma 2.7. *A surjective function $f : X \to Y$ is pre-$\pi g\gamma$-closed (resp. regular $\pi g\gamma$-closed) if and only if for each subset B of Y and each $U \in \gamma O(X)$ (resp. $U \in BR(X)$) containing $f^{-1}(B)$, there exists a $\pi g\gamma$-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$.*

Corollary 2.8. *If a surjective function $f : X \to Y$ is pre-$\pi g\gamma$-closed (resp. regular $\pi g\gamma$-closed), then for each π-closed set K of Y and each $U \in \gamma O(X)$ (resp. $U \in BR(X)$) containing $f^{-1}(K)$, there exists $V \in \gamma O(Y)$ containing K such that $f^{-1}(V) \subseteq U$.*

Proof. Suppose that $f : X \to Y$ is pre-$\pi g\gamma$-closed (resp. regular $\pi g\gamma$-closed). Let K be any π-closed set of Y and $U \in \gamma O(X)$ (resp. $U \in BR(X)$) containing $f^{-1}(K)$. By Lemma 2.7, there exists a $\pi g\gamma$-open set G of Y such that $K \subseteq G$ and $f^{-1}(G) \subseteq U$. Since K is π-closed, by Lemma 1.1, $K \subseteq \gamma int(G)$. Put $V = \gamma int(G)$. Then, $K \subseteq V \in \gamma O(Y)$ and $f^{-1}(V) \subseteq$

U. □

Definition 2.9 ([10]). *A function $f : X \to Y$ is said to be*

π-irresolute if $f^{-1}(F)$ is π-closed in X for every π-closed set F of Y.

m-π-closed if $f(F)$ is π-closed in Y for every π-closed set F of X.

Lemma 2.10. *A function $f : X \to Y$ is π-irresolute if and only if $f^{-1}(F)$ is π-open in X for every π-open set F of Y.*

Theorem 2.11. *If $f : X \to Y$ is π-irresolute pre-$\pi g\gamma$-closed bijection, then f(H) is $\pi g\gamma$-closed in Y for each $\pi g\gamma$-closed set H of X.*

Proof. Let H be any $\pi g\gamma$-closed set of X and V an π-open set of Y containing f(H). Since $f^{-1}(V)$ is an π-open set of X containing H, $\gamma cl(H) \subseteq f^{-1}(V)$ and hence $f(\gamma cl(H)) \subseteq V$. Since f is pre-$\pi g\gamma$-closed and $\gamma cl(H) \in \gamma C(X)$, $f(\gamma cl(H))$ is $\pi g\gamma$-closed in Y. We have $\gamma cl(f(H)) \subseteq \gamma cl(f(\gamma cl(H))) \subseteq V$. Therefore, f(H) is $\pi g\gamma$-closed in Y.□

Definition 2.12. *A function $f : X \to Y$ is said to be pre-$\pi g\gamma$-continuous or γ-$\pi g\gamma$-continuous if $f^{-1}(K)$ is $\pi g\gamma$-closed in*

X for every $K \in \gamma C(Y)$.

It is obvious that a function $f : X \to Y$ is pre-$\pi g\gamma$-continuous if and only if $f^{-1}(V)$ is $\pi g\gamma$-open in X for every $V \in \gamma O(Y)$.

Theorem 2.13. *If $f : X \to Y$ is m-π-closed pre-$\pi g\gamma$-continuous bijection, then $f^{-1}(K)$ is $\pi g\gamma$-closed in X for each $\pi g\gamma$-closed set K of Y.*

Proof. Let K be $\pi g\gamma$-closed set of Y and U an π-open set of X containing $f^{-1}(K)$. Put $V = Y - f(X - U)$, then V is an π-open in Y, $K \subseteq V$ and $f^{-1}(V) \subseteq U$. Therefore, we have $\gamma cl(K) \subseteq V$ and hence $f^{-1}(K) \subseteq f^{-1}(\gamma cl(K)) \subseteq f^{-1}(V) \subseteq U$. Since f is pre-$\pi g\gamma$-continuous and $\gamma cl(K)$ is γ-closed in Y, $f^{-1}(\gamma cl(K))$ is $\pi g\gamma$-closed in X and hence $\gamma cl(f^{-1}(K)) \subseteq \gamma cl(f^{-1}(\gamma cl(K))) \subseteq U$. This shows that $f^{-1}(K)$ is $\pi g\gamma$-closed in X.□

Recall that a function $f : X \to Y$ is said to be γ-irresolute [9] if $f^{-1}(V) \in \gamma O(X)$ for every $V \in \gamma O(Y)$.

Remark 2.14. *Every γ-irresolute function is pre-$\pi g\gamma$-continuous but not conversely.*

Proof. Let $A \in \gamma O(Y)$. Since f is γ-irresolute, $f^{-1}(A) \in \gamma O(X)$ and so, by Remarks 1.3 and 1.4, $f^{-1}(A)$ is $\pi g\gamma$-open in X. Hence f is pre-$\pi g\gamma$-continuous. □

Example 2.15. *Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is pre-$\pi g\gamma$-continuous but not γ-irresolute.*

Corollary 2.16. *If $f : X \to Y$ is m-π-closed γ-irresolute bijection, then $f^{-1}(K)$ is $\pi g\gamma$-closed in X for each $\pi g\gamma$-closed set K of Y.*

Proof. It is obtained from Theorem 2.13 and Remark 2.14. □

For the composition of pre-$\pi g\gamma$-closed functions, we have the following Theorems.

Theorem 2.17. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then the composition $gof : X \to Z$ is pre-$\pi g\gamma$-closed if f is pre-$\pi g\gamma$-closed and g is π-irresolute pre-$\pi g\gamma$-closed bijection.*

Proof. The proof follows immediately from Theorem 2.11. □

Theorem 2.18. *Let $f : X \to Y$ and $g : Y \to Z$ be functions and let the composition $gof : X \to Z$ be pre-$\pi g\gamma$-closed. Then the following hold:*

If f is a γ-irresolute surjection, then g is pre-$\pi g\gamma$-closed;

If g is a m-π-closed pre-$\pi g\gamma$-continuous injection, then f is pre-$\pi g\gamma$-closed.

Proof.

Let K $\in \gamma$C(Y). Since f is γ-irresoulte and surjective, f^{-1}(K) $\in \gamma$C(X) and (gof)(f^{-1}(K)) = g(K). Therefore, g(K) is $\pi g\gamma$-closed in Z and hence g is pre-$\pi g\gamma$-closed.

Let H $\in \gamma$C(X). Then (gof)(H) is $\pi g\gamma$-closed in Z and g^{-1}((gof)(H)) = f(H). By Theorem 2.13, f(H) is $\pi g\gamma$-closed in Y and hence f is pre-$\pi g\gamma$-closed.

□

Lemma 2.19. *A surjective function $f : X \to Y$ is almost $\pi g\gamma$-closed if and only if for each subset B of Y and each $U \in RO(X)$ containing $f^{-1}(B)$, there exists a $\pi g\gamma$-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$.*

Corollary 2.20. *If a surjective function $f : X \to Y$ is almost $\pi g\gamma$-closed, then for each π-closed set K of Y and each $U \in RO(X)$ containing $f^{-1}(K)$, there exists $V \in \gamma O(Y)$ such that $K \subseteq V$ and $f^{-1}(V) \subseteq U$.*

Proof. The proof is similar to that of Corollary 2.8. □

Recall that a topological space (X, τ) is said to be quasi-normal [23] if for every disjoint π-closed sets A and B of X, there exist disjoint sets U, V $\in \tau$ such that A $\subseteq$ U and B $\subseteq$ V.

Definition 2.21. *A topological space (X, τ) is said to be quasi-γ-normal if for every disjoint π-closed sets A and B of X, there exist disjoint sets $U, V \in \gamma O(X)$ such that $A \subseteq U$ and $B \subseteq V$.*

Theorem 2.22. *Let $f : X \to Y$ be a π-irresolute almost $\pi g\gamma$-closed surjection. If X is quasi-normal, then Y is quasi-γnormal.*

Proof. Let K_1 and K_2 be any disjoint π-closed sets of Y. Since f is π-irresolute, $f^{-1}(K_1)$ and $f^{-1}(K_2)$ are disjoint π-closed sets of X. By the quasi-normality of X, there exist disjoint open sets U_1 and U_2 such that $f^{-1}(K_i) \subseteq U_i$, where i=1,2. Now, put G_i = int(cl(U_i)) for i=1, 2, then $G_i \in$ RO(X), $f^{-1}(K_i) \subseteq U_i \subseteq G_i$ and $G_1 \cap G_2 = \emptyset$. By Corollary 2.20, there exists $V_i \in \gamma$O(Y) such that $K_i \subseteq V_i$ and $f^{-1}(V_i) \subseteq G_i$, i= 1, 2. Since $G_1 \cap G_2 = \emptyset$, f is surjective we have $V_1 \cap V_2 = \emptyset$. This

shows that Y is quasi-γ-normal. □

Definition 2.23 ([8]). *A function $f : X \to Y$ is said to be γ-open (resp. γ-closed), if $f(U) \in \gamma O(Y)$ (resp. $f(U) \in \gamma C(Y)$) for every open (resp. closed) set U of X.*

Definition 2.24 ([8]). *A function $f : X \to Y$ is said to be $g\gamma$-closed if $f(U)$ is $g\gamma$-closed in Y for every closed set U of X.*

The following four Corollaries are immediate consequences of Theorem 2.22.

Corollary 2.25. *If $f: X \to Y$ is a π-irresolute $\pi g\gamma$-closed surjection and X is quasi-normal, then Y is quasi-γ-normal.*

Corollary 2.26. *If $f: X \to Y$ is a π-irresolute $g\gamma$-closed surjection and X is quasi-normal, then Y is quasi-γ-normal.*

Corollary 2.27. *If $f: X \to Y$ is a π-irresolute γ-closed surjection and X is quasi-normal, then Y is quasi-γ-normal.*

Corollary 2.28. *If $f: X \to Y$ is a π-irresolute closed surjection and X is quasi-normal, then Y is quasi-γ-normal.*

Definition 2.29 ([8]). *A function $f: X \to Y$ is said to be strongly γ-closed (resp. strongly γ-open) if for each $F \in \gamma C(X)$ (resp. $F \in \gamma O(X)$), $f(F) \in \gamma C(Y)$ (resp. $f(F) \in \gamma O(Y)$).*

Remark 2.30. *Every strongly γ-closed function is γ-closed but not conversely.*

Proof. Let A be a closed set of X. Then A is γ-closed set of X. Since f is strongly γ-closed, $f(A) \in \gamma C(Y)$. Hence f is γ-closed. □

Example 2.31. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is γ-closed but not strongly γ-closed.*

Theorem 2.32 ([18]). *Let A be a subset of a topological space X. Then*

$A \in BO(X)$ if and only if $bcl(A) \in BR(X)$.

$A \in BC(X)$ if and only if $bint(A) \in BR(X)$.

Theorem 2.33. *Let $f: X \to Y$ be a π-irresolute regular $\pi g\gamma$-closed surjection. If X is quasi-γ-normal, then Y is quasi-γnormal.*

Proof. Although the proof is similar to that of Theorem 2.22, we will state it for the convenience of the reader. Let K_1 and K_2 be any disjoint π-closed sets of Y. Since f is π-irresolute, $f^{-1}(K_1)$ and $f^{-1}(K_2)$ are disjoint π-closed sets of X. By the quasi-γ-normality of X, there exist disjoint sets $U_1, U_2 \in \gamma O(X)$ such that $f^{-1}(K_i) \subseteq U_i$, for i=1,2. Now, put $G_i = \gamma cl(U_i)$ for i=1, 2, then by Theorem 2.32, $G_i \in BR(X)$, $f^{-1}(K_i) \subseteq U_i \subseteq G_i$ and $G_1 \cap G_2 = \emptyset$. By Corollary 2.8, there exists $V_i \in \gamma O(Y)$ such that $K_i \subseteq V_i$ and $f^{-1}(V_i) \subseteq G_i$, where i= 1, 2. Since f is surjective and $G_1 \cap G_2 = \emptyset$, we obtain $V_1 \cap V_2 = \emptyset$. This shows that Y is quasi-γ-normal. □

Corollary 2.34. *Let $f: X \to Y$ be a π-irresolute pre-$\pi g\gamma$-closed surjection. If X is quasi-γ-normal, then Y is quasi-γnormal.*

Remark 2.35. *Every strongly γ-closed function is pre-$\pi g\gamma$-closed but not conversely.*

Proof. Let $F \in \gamma C(X)$. Since f is strongly γ-closed, $f(F) \in \gamma C(Y)$ and so, by Remarks 1.3 and 1.4, f(F) is $\pi g\gamma$-closed in Y. Hence f is pre-$\pi g\gamma$-closed. □

Example 2.36. *Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\sigma = \{\emptyset, Y, \{b, d\}, \{a, b, d\}\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is pre-$\pi g\gamma$-closed but not strongly γ-closed.*

Corollary 2.37. *If $f: X \to Y$ is a π-irresolute strongly γ-closed surjection and X is quasi-γ-normal, then Y is quasi-γnormal.*

Theorem 2.38. *Let $f: X \to Y$ be a m-π-closed pre-$\pi g\gamma$-continuous injection. If Y is quasi-γ-normal, then X is quasi-γnormal.*

Proof. Let H_1 and H_2 be any disjoint π-closed sets of X. Since f is a m-π-closed injection, $f(H_1)$ and $f(H_2)$ are disjoint π-closed sets of Y. By the quasi-γ-normality of Y, there exist disjoint sets $V_1, V_2 \in \gamma O(Y)$ such that $f(H_i) \subseteq V_i$, for i=1,2.

Since f is pre-$\pi g\gamma$-continuous $f^{-1}(V_i)$ and $f^{-1}(V_i)$ are disjoint $\pi g\gamma$-open sets of X and $H_i \subseteq f^{-1}(V_i)$ for i=1,2. Now, put $U_i = \gamma int(f^{-1}(V_i))$ for i=1,2. Then $U_i \in \gamma O(X)$, $H_i \subseteq U_i$ and $U_1 \cap U_2 = \emptyset$. This shows that X is quasi-γ-normal. □

Corollary 2.39. *If $f: X \to Y$ is a m-π-closed γ-irresolute injection and Y is quasi-γ-normal, then X is quasi-γ-normal.*

Proof. This is an immediate consequence of Theorem 2.38, since every γ-irresoulte function is pre-$\pi g\gamma$-continuous. □

Definition 2.40. *A topological space X is said to be quasi-regular if for each π-closed set F and each point $x \in X-F$, there exist disjoint U, $V \in \tau$ such that $x \in U$ and $F \subseteq V$.*

Theorem 2.41. *For a topological space X, the following properties are equivalent:*

X is quasi-regular;

For each π-open set U in X and each $x \in U$, there exists $V \in \tau$ such that $x \in V \subseteq cl(V) \subseteq U$;

For each π-open set U in X and each $x \in U$, there exists a clopen set V such that $x \in V \subseteq U$.

Proof. (1) ⇒ (2): Let U be an π-open set of X containing x. Then X\U is a π-closed set not containing x. By (1), there exist disjoint X\cl(V), $V \in \tau$ such that $x \in V$ and X\U ⊆ X\cl(V). Then we have $V \in \tau$ such that $x \in V \subseteq cl(V) \subseteq U$.

⇒ (3): Let U be an π-open set of X containing x. By (2), there exists $V \in \tau$ such that $x \in V \subseteq cl(V) \subseteq U$. Take V = cl(V). Thus V is closed and so V is clopen. Hence we have V is clopen set such that $x \in V \subseteq U$.

⇒ (1): Let F = X\U be a π-closed set not containing x. Then U is an π-open set of X containing x. By (3), there exists a clopen set V such that $x \in V \subseteq U$. Then there exist disjoint G = X\V, $V \in \tau$ such that $x \in V$ and F = X\U ⊆ G = X\V.

Hence X is quasi-regular.□

Definition 2.42. *A topological space X is said to be quasi-γ-regular if for each π-closed set F and each point $x \in X-F$, there exist disjoint U, $V \in \gamma O(X)$ such that $x \in U$ and $F \subseteq V$.*

Theorem 2.43. *For a topological space X, the following properties are equivalent:*

X is quasi-γ-regular;

For each π-open set U in X and each $x \in U$, there exists $V \in \gamma O(X)$ such that $x \in V \subseteq \gamma cl(V) \subseteq U$;

For each π-open set U in X and each $x \in U$, there exists $V \in BR(X)$ such that $x \in V \subseteq U$.

Theorem 2.44. *Let $f : X \to Y$ be a π-irresolute γ-open almost $\pi g\gamma$-closed surjection. If X is quasi-regular, then Y is*

quasi-γ-regular.

Proof. Let $y \in Y$ and V be an π-open neighbourhood of y. Take a point $x \in f^{-1}(y)$. Then $x \in f^{-1}(V)$ and $f^{-1}(V)$ is π-open in X. By the quasi-regularity of X, there exists an π-open set U of X such that $x \in U \subseteq cl(U) \subseteq f^{-1}(V)$. Then $y \in f(U) \subseteq f(cl(U)) \subseteq V$. Also, since U is open set of X and f is γ-open, $f(U) \in \gamma O(Y)$. Moreover, since U is β-open, by Theorem

1.6, cl(U) is regular closed set of X. Since f is almost $\pi g\gamma$-closed, f(cl(U)) is $\pi g\gamma$-closed in Y. Therefore, we obtain $y \in f(U) \subseteq \gamma cl(f(U)) \subseteq \gamma cl(f(cl(U))) \subseteq V$. It follows from Theorem 2.43 that Y is quasi-γ-regular. □

Corollary 2.45. *If $f : X \to Y$ is a π-irresolute γ-open $\pi g\gamma$-closed surjection and X is quasi-regular, then Y is quasi-γ-*

regular.

Corollary 2.46. *If $f : X \to Y$ is a π-irresolute γ-open γ-closed surjection and X is quasi-regular, then Y is quasi-γ-regular.*

Theorem 2.47. *Let $f : X \to Y$ be a π-irresolute strongly γ-open regular $\pi g\gamma$-closed surjection. If X is quasi-γ-regular, then Y is quasi-γ-regular.*

Proof. Let F be any π-closed set of Y and $y \in Y - F$. Then $f^{-1}(F)$ is π-closed in X and $f^{-1}(F) \cap f^{-1}(y) = \emptyset$. Take a point $x \in f^{-1}(y)$. Since X is quasi-γ-regular, there exist disjoint sets $U_1, U_2 \in \gamma O(X)$ such that $x \in U_1$ and $f^{-1}(F) \subseteq U_2$. Therefore, we have $f^{-1}(F) \subseteq U_2 \subseteq \gamma cl(U_2)$, by Theorem 2.32, $\gamma cl(U_2) \in BR(X)$ and $U_1 \cap \gamma cl(U_2) = \emptyset$. Since f is regular $\pi g\gamma$-closed, by Corollary 2.8, there exists $V \in \gamma O(Y)$ such that $F \subseteq V$ and $f^{-1}(V) \subseteq \gamma cl(U_2)$. Since f is strongly γ-open, we have $f(U_1) \in \gamma O(Y)$. Moreover, $U_1 \cap f^{-1}(V) = \emptyset$ and hence $f(U_1) \cap V = \emptyset$. Consequently, we obtain $y \in f(U_1) \in \gamma O(Y)$, $F \subseteq V \in \gamma O(Y)$ and $f(U_1) \cap V = \emptyset$. This shows that Y is quasi-γ-regular.□

Corollary 2.48. *If $f : X \to Y$ is a π-irresolute strongly γ-open pre-πgγ-closed surjection and X is quasi-γ-regular, then Y is quasi-γ-regular.*

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