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Combinatorial inertia classes that represent stars Ncωn∗ complete one to one semi groups in a three-dimensional model

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Abstract

Xn, where n is a number between 1 and n, must be distinct and non-negative. The 3D Nildempotency star-like classes Ncω∏ partial one-one were examined with the help of the star-like operator |αwi −αwi+1| + |α∏wi − wi+1| ≤ |wi − wi+1| + |α∏wi+1 − wi|. Additionally, a conventional rectangular A4 sheet of paper was used to create the geometric model of a three-dimensional star-like nildempotency transformation semigroup.For every star-shaped polygon in Ncω∏ with n star-shaped vertices V ∗, star-shaped edges E∗, star-shaped faces F ∗, and n star-shaped angles α∏, we prove that V ∏ − E∏ + F ∏ = 2.

Keywords: Cobinatorial composite function, Geometry, NIldempotency, Semigroup, Star-like.

1 Introduction

Although semigroup theory has attracted a lot of attention from scholars, Akinwunmi's recently created star-like transformation semigroup has received comparatively less attention [1]. According to Akin-Wunmi, a star-like semigroup S∏ with empty mapping is nildempotent if and only if there exists a vector V such that St =, where t, k : x1, x2, x3,..., xn = for every x1, x2, x3,... S implies α S where ek D(α) : ek = e. The nildempotency degree of S■, represented by ND(S■), is the smallest element t, k V : St = = ek = e if S■ is nildempotent. It is necessary to discover the general relation, which in turn emphasizes its applications to Mathematics and Science in general, after we notice that the nildempotent elements constitute a distinct sub-semigroup class. The combinatorial character of the numerical sequence and its triangular arrangement follow naturally from this.
If the (α, β, μ):(α ∏ β) ∏ μ=α ∏ (β ∏ μ) transformation semigroup is associative, then the inverse or partial one to one transformation semigroup, which is given by Xn = 1, 2, 3,..., n, where α:Dom α = Xn, is also known as the semigroup.

Inverse symmetric semigroup, monoid, and semigroup operator with composition of mappings are other names for it. Another semigroup in the one-one transformation semigroup is the star-like partial one-one transformation semigroup, which is represented as Iαω∗ in In.
Although mapping is another word for transformation, the former is more often used. Look at Bhattacharya et al. [3] and Howie [4] for further details on semigroup of mappings.

In accordance with Malik [5] and references [?,?] for the research and definition of the partial transformation semigroup, the domain and image set of every given transformation $\alpha\prod \in \alpha\omega\prod$ were represented as $D(\alpha\prod)$ and $I(\alpha\prod)$ accordingly. When $c+(\alpha*) = |\ t\alpha-1$, it is claimed that the star-like transformation semigroup satisfies the collapse function.
The relapse function is represented as $r+(\alpha) = |\ n - c+(\alpha*)\ |$, where n is an element of N, and t is a vector in $T\alpha\omega\prod$.

2 Preliminary Notes

2.1 Generalization of 3 Dimensional Star-like Sequence through Some Combinatorial Composite Functions

Let $X = \{w_1, w_2, w_3, \ldots w_n\}$ be a distinct finite n-elements set where $N_c\omega^*_n$ is a 3 Dimensional star-like partial one to one classical semigroup such that

$$|\alpha w_i - \alpha w_{i+1}| + |\alpha\ w_i{}^* - w_{i+1}| \leq |w_i - w_{i+1}| + |\alpha\ w_{i+1} - w_i|^* \tag{2.1}$$

for all $w_i, w_{i+1} \in D(\alpha^*)$ and $\alpha^* w_i, \alpha^* w_{i+1} \in I(\alpha^*)$ where $i \geq n \geq 1$; $n \in$ N, then we give a transformation of X in the form:

$$\alpha^*_n = \begin{matrix} w_1 & w_2 & w_3 & \ldots & w_n \\ \alpha^* w_1 & \alpha^* w_2 & \alpha^* w_3 & \ldots & \alpha^* w_n \end{matrix} \tag{2.2}$$

The element $\alpha^* w_i$ is call the value of points in the image of the transformation of α^* and w_i is call the value of points in the domain of the transformation of α^*. The fact that $N_c\omega^*_n$ is a 3 Dimensional star-like transformation implies that X can be written as $\alpha^* : X \longrightarrow X$. Then, the transformation of X is a function $\alpha^* : B \longrightarrow X$, where $B = \{f_1, f_2, f_3, \ldots f_n\}$ is a subset of X then we write the element of α^* such that:

$$\alpha^*_n = \begin{matrix} f_1 & f_2 & f_3 & \ldots & f_n \\ \alpha^* f_1 & \alpha^* f_2 & \alpha^* f_3 & \ldots & \alpha^* f_n \end{matrix} \tag{2.3}$$

Then the set of all 3 dimensional star-like one to one (inverse) transformation of X is denoted by $N_c\omega^*_n$. We can rewrite the element of α^* in equation (1) in the form:

$$\alpha^*_n = \begin{matrix} 1 & 2 & 3 & \ldots & n \\ k_1 & k_2 & k_3 & \ldots & k_n \end{matrix} \tag{2.4}$$

here $k_i = \alpha^* \omega_i \in I(\alpha^*)$ and $n \in$ N. By adopting the methodology given by Akinwunmi et al. [2] on Nildempotency of partial one to one contraction Cl_n transformation semigroups, we obtained the 3 dimensional Nildempotency star-like classes $N_c\omega^*$ partial one to one transformation semigroups. The star-like nildempotency $N_c\omega^*$ degree of order 2^n were generated from Equation (1): when $n = 1$

$$N_c\omega^* = \begin{matrix} 1 \\ 1 \end{matrix}^2 \qquad \begin{matrix} 1 \\ 0 \end{matrix}$$

when $n = 2$

$$N_c\omega^* = \begin{matrix} 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 2 & 1 & 2 & 2 \end{matrix}^2$$

when $n = 3$

$$C\omega^* = \begin{matrix} 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \\ 1 & 2 & 3 & 3 & 2 & 1 & 2 & 1 & 0 \end{matrix}^n$$

2.2 Geometric Model of 3 Dimensional Nildempotency star like classes $N_c\omega^*_n$ partial one to one transformation semigroups.

Figure 1 is the result of creating a geometric model of three-dimensional nildempotency star-like transformation semigroups using the star-like folding principle on a regular rectangular A4 sheet of paper, starting from the top right corner and working our way down.

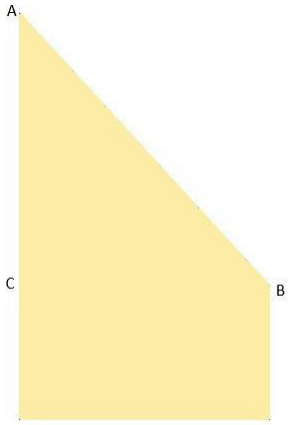


Figure 1: 3 Dimensional Nildempotency Star-like Trapezium

Then, Cutting out the remaining sheet below vertices C and B in Figure 1: we obtained Figure 2:

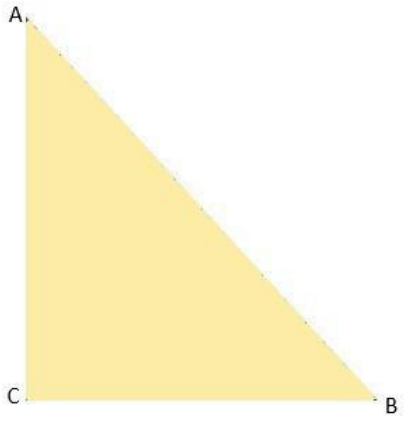


Figure 2: 3 Dimensional Nildempotency Star-like Right Angle Triangle

From the star-like folding principle, holding vertex of A and joining it to the vertex of B in Figure 2: we obtain Figure 3: below

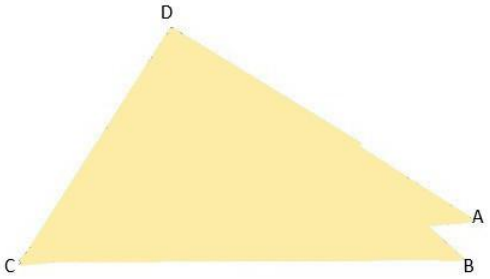


Figure 3: 3 Dimensional Nildempotency Star-like Equilateral Triangle

Now, unfolding the Figure 1:, Figure 2: and Figure 3: above, we reverse the structure to a standard plain sheet with some star-like paths retained, we generated the star-like nildempotency classes of semigroup as shown in Figure 4:

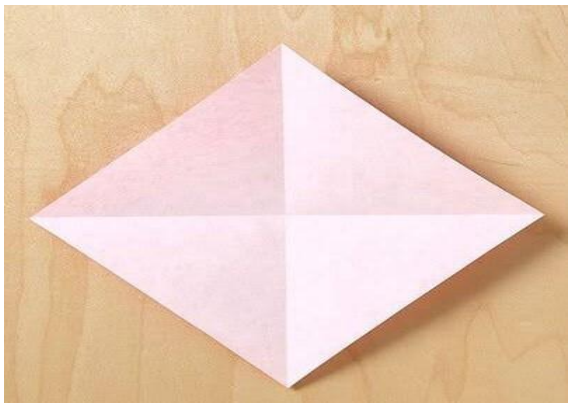


Figure 4: 3 Dimensional Nildempotency Star-like Square Pyramid

Holding the vertices of A and B of the Figure 4: in such a way that the vertices of C and D will facing 'you' directly there by joining the vertices of A and B together allowing the middle of vertices A and B to be folded inward to obtain Figure 5:

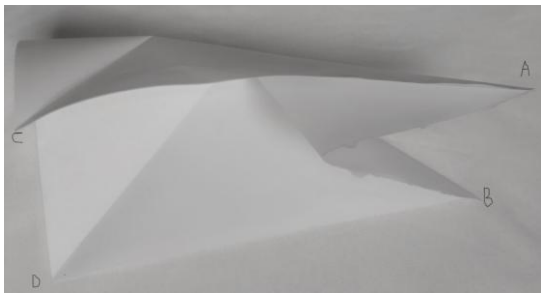


Figure 5: 3 Dimensional Nildempotency Star-like Polygon

Repeating the procedure in Figure 5: of vertices C and D, we obtained Geometric model of 3 Dimensional Nildempotency Star-like Classes $N_e\omega^*_n$ partial one-one transformation semigroups represented in Figure 6:



Figure 6: Bottom View of 3 Dimensional Nildempotency Star-like Pyramid

The star-like 3 Dimensional model in Figure7: represent the star-like Nildempotency rectangular prism with the composition of:

i Star-like Faces F^*

ii Star-like Edges E^*

iii Star-like Vertices V^*

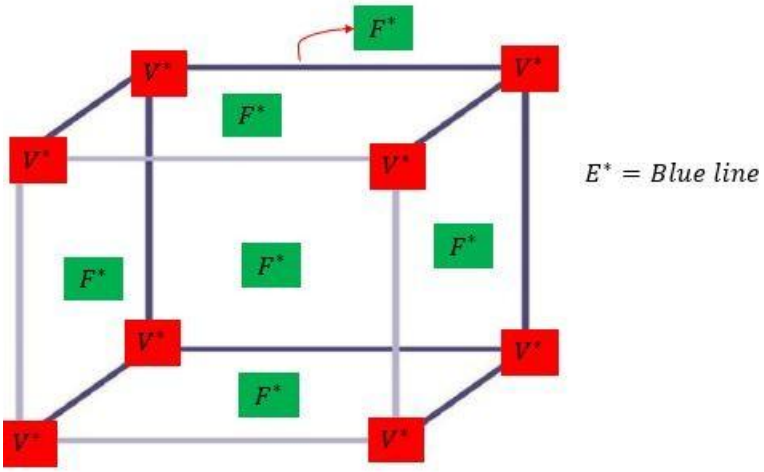


Figure 7: Star-like Nildempotency Square Prism

By the star-like folding principle structure we unfold the Fig 7: to obtained the general 3 Dimensional star-like Nildempotency equation.

$$F^* + V^* = E^* + 2 \tag{2.5}$$

which is a relation to the unfolded 3 Dimensional star-like Nildempotency rectangular prism. Therefore to obtained the volume of a 3 Dimensional star-like Nildempotency triangular prism, we must begin to construct a star-like triangular path with a 3 Dimensional star-like Nildempotency array of a control star-like Nildempotency disk point which form an n sided star-like Nildempotency 3 Dimensional depths. From equation 1 combining with 3 Dimensional general star-like Nildempotency, we obtained

Some notable notation used in this paper

$N_c\omega^*_n$	3 Dimensional Nildempotency star-like transformation
ϕ^*	Star-like nildempotency disk constant point
ϕ^*_n	Set of all star-like disk constant relation
C^*	Star-like convex polygon
R^*_n	Star-like polygon
$M(\alpha^*)$	Star-like Maximum element
$\phi^* = 2$	Inner star-like nildempotency disk constant

3 Main Results

These are the main results of the paper.

Given any star-like transformation α^* in partial one-one star-like semigroup $N_c\omega^*_n$ then the following statement are equivalent:

i Any $\alpha \in N_c\omega^*_n$ has a maximum element $m(\alpha^*)$

ii $|N_c\omega^*_n| = \sum_{q=2}^{n} \begin{matrix} n + 2q^* \\ 3q^* \end{matrix}$

iii $F(n, c^*, m^*) = 2^{(n-c^*)+1} - 1 \ : n, m^* \geq q^*$

Proof. $(i) \longrightarrow (ii)$
Suppose $F(n, c^*, m^*)$ is a Nildempotency star-like composition combinatorial function, let $N_i = \{i, i+1, i+2, \ldots\}, i = \{0, 1, 2, \ldots\}$ be a distinct non negative integer such that $N_{q^*}\omega^*$ contain a
star-like transformation $\alpha^* \in N_c\omega^*_n$ then $i \in N_i$ of $Dam(\alpha^*)$ can be chosen from

N_i in $\begin{matrix} n \\ q^* \end{matrix}$ ways.

If α^* is a bijective map and $N_c\omega^*_n$ is reducible. $Im(\alpha^*) = 0$
if $|q^*(\alpha^*)| = 1$ but if

$$n = q^*; |q^*(\alpha^*)| = -1$$

for each value of $\alpha \in N_c\omega^*_n$ such that

$$|N_c\omega_n| = \begin{matrix} * \\ \end{matrix} \begin{matrix} n + 2q^* \\ 3q^* \end{matrix}$$

$(ii) \longrightarrow (iii)$ Since $\alpha \in N_c\omega^*$ is star-like Nildempotency , there exist finitely many element such that $Dm(\alpha^*) = Im(\alpha^*) where \lambda^{*n}_{r_n}$ form a total of

$$\sum_{n=1}^{2^{(n-q)+1}} \frac{n+2q^{k}}{2^{(n-m)+1}-1} \; 3q^{*} = F(n,q,m)$$

$(iii) \longrightarrow (i)$ Suppose $m(\alpha^{*})$. denote maximum element in $Im(\alpha^{*})$ and $N_c\omega^{*}_{n} \subseteq \alpha\omega^{*}_{n}$, then consider a mapping

$$r^{*} = \begin{matrix} u_1 & u_2 & u_0 \\ m_1 & m_2 & m_0 \end{matrix} \in m(\alpha)$$

such that there exist another element $\lambda^{*} \in m(\alpha$ with $r^{*} \leq \lambda^{*}$and$I_0 < \lambda^{*}$. since r and λ_0 are bijective we have

$$(rI\omega^{*}) \setminus (\lambda_0 I\omega^{*}) = m(\alpha^{*})_{n} \quad _{n}$$

□

Suppose $N_c\omega^{n}$ is a set of star-like Nildempotency finite transformation semigroup with a star-like composite relation,

$$a^{*}_{i} + b^{*}_{j} + c^{*}_{k} = N_c\omega^{*}_{n} \tag{3.1}$$

Such that for any Star-like nildempotency semigroup, there exist a star-like operator $\phi^{*} = 2$, then by Star-like general 3 Dimensional Nildempotency equation and operator

$$F^{*} + V^{*} = E^{*} + 2 \tag{3.2}$$

$$|N_c\omega_{n^{*}}| = |\alpha_{i+1} - \alpha_i| \leq |\alpha_i - \omega_i| \tag{3.3}$$

we have that

$$\frac{1}{2}V^{*} = \alpha\omega_i \quad | \quad -\omega_{i+1} \quad | \leq |\alpha_i\omega_i \quad -\omega_i| \tag{3.4}$$

Since $N_c\omega^{*}_{n}$ satisfy (7), it shows that

$$\frac{1}{2}V^{*} = \frac{E^{*}-F^{*}}{2} + \phi^{*} \tag{3.5}$$

Thus; $\frac{1}{2}V^*(N_c\omega_n^*)$ implies $V^* = E^* - F^* + \phi^*$ which is the required Star-like Nildempotency vertices of any $\alpha_n^* \in N_c\omega_n^*$ transformation semigroup with a star-like Nildempotency disk constant point ϕ^*.

□

Example 1 Given $\alpha^* = \begin{matrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & \emptyset \end{matrix}$; such that $\alpha^* \in N_c\omega^*$ with star-like Nildempotency constant point $\phi^* \in \alpha^*$. Then the folded $\alpha^* \in N_c\omega^*$ is illustrated in figure 8

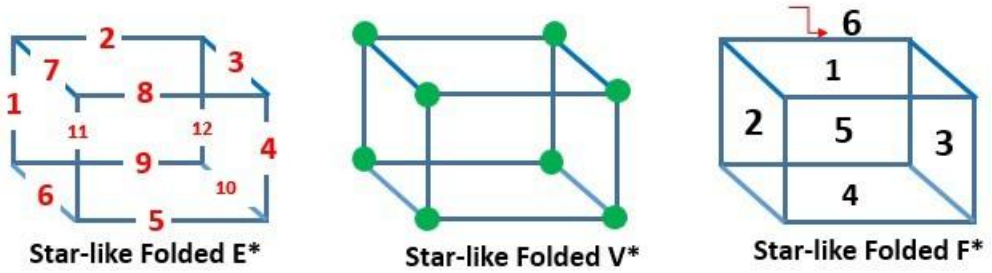


Figure 8: Star-like Nildempotency cube

obtain the star-like structure general Nildempotency structure of $\alpha^* \in N_c\omega^*$;
solution:
From equation (9) and (12) we can obtain $F^* + V^* = E^* + 2$ with a star-like Nildempotency constant $\phi^* \in \alpha^*$, Then by star-like folding principle, unfolding the figure(Star- like Nildempotency cubes) we have that $F^* = 6$; $V^* = 8$ implies $6 + 8 = E + 2$ implies $E^* = 12$

Given a star-like nildempotency transformation $N_c\omega^*$, Let R^* be a star-like polygon with n star-like vertices and n sides with star-like interior angles, $\alpha_i^*, \ldots \alpha_n^*$. Then

$$Area(R^*) = \left(\sum_{n=1}^{\infty} \alpha_n\right) - (n-2)\pi$$

Proof. Let C^* be a star-like convex polygon in R^*, this can be done rigorously by arranging C^* so that the origin lies in the interior of C^* and projecting the boundary of C^* on S^2 using the function

$$f(a, b, c) = \frac{(a, b, c)}{\sqrt{a^2 + b^2 + c^2}}$$

It's easy to check that vertices of ϕ_n^* go to part on S^2, edges go to part of great circles (ϕ^*) and faces go to polygon.

Let V^*, E^* and F^* denote the number of star-like vertices, edges, and faces of C^* respectively. Let $R_1^* + \cdots R_n^*$ be the polygon on S^2. Since their union is S^2

$$Area(R^*) + Area(R^*) + \cdots + Area(R^*) = Area(S^2)$$

Let n_i be the number of edges of R_i and $\alpha_{ij} for j = 1, \ldots n_i$ be the interior angles. Then by theorem 3.2, we have

$$\frac{1}{2}V^* = \frac{E^* - F^*}{2} + \phi^* \quad where\ \phi^* = 2$$

implies

$$V^* = E^* - F^* + 4$$

since $n_i \in R_i$ and α_{ij} for $j = 1, \cdots n_i$ then

$$\left(\sum_{n}\sum_{n_i} \alpha_{ij} - n_i\pi + 2\pi\right) = 4\pi$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i}\alpha_{ij} - \sum_{i=1}^{n} n_i\pi + \sum_{i=1}^{n} 2\pi = 4\pi$$

Since every edges is shared by two star-like polygon

$$\sum_{i=1}^{n} n_i\pi = 2\pi E^*$$

Also, since the sum of angles at every vertex is 2π

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i}\alpha_{ij} = 2\pi V^*$$

Hence

$$2\pi V^* - 2\pi E^* + 2\pi F^* = 4\pi$$

which is equivalent to $V^* - E^* + F^* = 2$ □

4 Discussion and Conclusion

Given a star-like nildempotency transformation semigroup $N_c\omega^*_n$, Let R^*_n be a star-like polygon such that $R^*_n \in N_c\omega^*_n$ which satisfies the following:

i Any two star-like vertices of R^*_n can be connected by chain of star-like edges.

ii Any star-like loop on R^*_n which made up of star-like straight line segment then $V^* - E^* + F^* = 2$ for all $R^*_n \in N_c\omega^*_n$

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